B

$$\lim \frac{\left(\frac{6}{5}\right)^{2n} + 2\left(\frac{3}{2}\right)^{n+1}}{\left(\frac{9}{6}\right)^{n} - \left(\frac{11}{10}\right)^{2n-1}} = \lim \frac{\left(\frac{36}{25}\right)^{n} + \cancel{2}\cdot\frac{3}{\cancel{2}}\left(\frac{3}{2}\right)^{n}}{\left(\frac{3}{2}\right)^{n} - \frac{10}{11}\cdot\left(\frac{121}{100}\right)^{n}} =$$

úprava 1b

$$= \lim \frac{\cancel{\left(\frac{3}{2}\right)^{n}}\left[\overbrace{\left(\frac{2}{3}\cdot\frac{36}{25}\right)^{n}}^{\to 0} + 3\right]}{\cancel{\left(\frac{3}{2}\right)^{n}}\left[1 - \frac{10}{11}\underbrace{\left(\frac{2}{3}\cdot\frac{121}{100}\right)^{n}}_{\to 0}\right]} = \frac{0+3}{1-0} = 3$$

dopočítání 1b

$$\frac{36}{25} < \frac{37{,}5}{25} = \frac{3}{2}$$

vytknutí dom. členu 1b

LIM PŘECH

$\boxed{3b}$

2. $f(x) = \frac{1}{2}x^2 - 4x + 6$, $x_0 = 1$ B

TEČNA:

$f'(x) = x - 4$ 0.4b

$f'(1) = 1 - 4 = -3$

$f(1) = \frac{1}{2} - 4 + 6 = 2,5 = \frac{5}{2}$

$\longrightarrow y = -3(x-1) + 2,5 = -3x + \underbrace{3 + 2,5}_{11/2} = -3x + 5,5$ 0.8b

Průsečíky tečny s osami: $P_y^T = [0; 5,5]$, $P_x^T = [\frac{11}{6}; 0]$ 0.2b

PARABOLA:

$P_y = [0; 6]$ 0.2b

P_x: $\frac{1}{2}x^2 - 4x + 6 = 0$

$x^2 - 8x + 12 = 0$

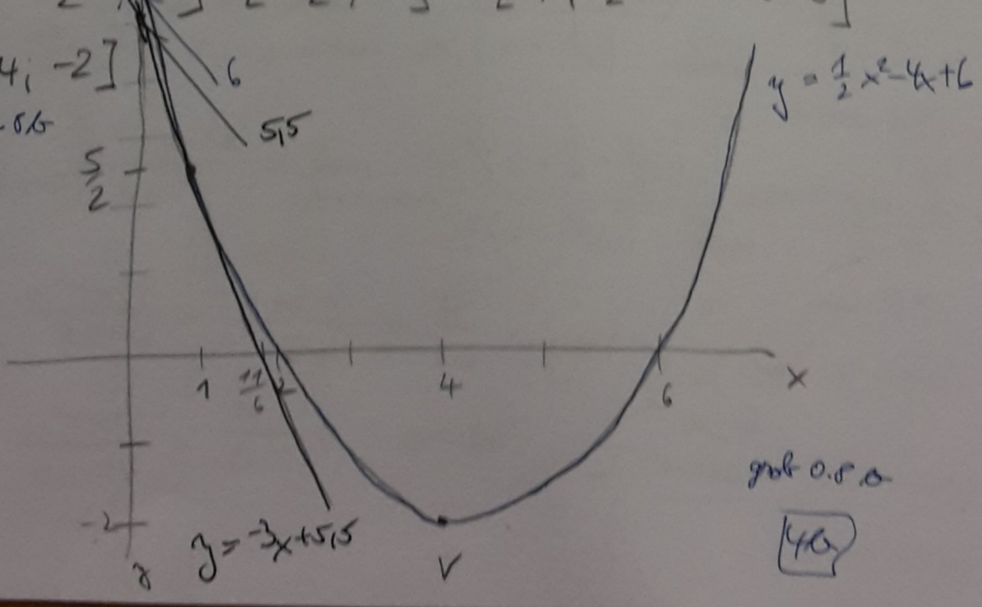
$x_1 = 6$

$x_2 = 2$ $\Rightarrow P_{x_1} = [6; 0]$, $P_{x_2} = [2; 0]$ 0.8b

$V = [\frac{x_1 + x_2}{2}; ?] = [\frac{6+2}{2}; ?] = [4; \frac{1}{2} \cdot 16 - 16 + 6] =$

$= [4; -2]$

0.8b



graf 0.8b

4b

B

3. $f(x) = \dfrac{x^2+x-2}{2x-4}$

1) $D_f = \mathbb{R} \setminus \{2\}$ $\frac{1}{4}$ b

2) SUDOST/LICHOST $\frac{1}{4}$ b

$f(1) = \dfrac{1+1-2}{2-4} = 0$

$f(-1) = \dfrac{1-1-2}{-2-4} = \dfrac{-2}{-6} = \dfrac{1}{3}$

$f(1) \neq f(-1) \Rightarrow$ f NENÍ SUDÁ

$f(1) \neq -f(-1) \Rightarrow$ f NENÍ LICHÁ

3) KLADNOST/ZÁPORNOST $\frac{1}{2}$ b

$x^2+x-2 = (x+2)(x-1)$

$x \in$	$(-\infty;-2)$	$(-2;1)$	$(1;2)$	$(2;+\infty)$
$f(x)$	$\frac{- \, -}{-}$ −	$\frac{+ \, -}{-}$ +	$\frac{+ \, +}{-}$ −	$\frac{+ \, +}{+}$ +

4) LIMITY V KRAJNÍCH BODECH D_f (celkem 1 b)

$$\lim_{x\to+\infty} \frac{x^2+x-2}{2x-4} = \lim_{x\to+\infty} \frac{x^{\cancel{2}}\left(1+\frac{1}{x}-\frac{2}{x^2}\right)}{\cancel{x}\left(2-\frac{4}{x}\right)} \underset{\text{LIM PŘECH}}{=} \frac{+\infty(1+0-0)}{2-0} = +\infty \quad \tfrac{1}{4}\text{ b}$$

$$\lim_{x\to-\infty} \frac{x^2+x-2}{2x-4} = \lim_{x\to-\infty} \frac{x^{\cancel{2}}\left(1+\frac{1}{x}-\frac{2}{x^2}\right)}{\cancel{x}\left(2-\frac{4}{x}\right)} \underset{\text{LIM PŘECH}}{=} \frac{-\infty(1-0-0)}{2-0} = -\infty \quad \tfrac{1}{4}\text{ b}$$

$$\lim_{x\to2^+} \frac{x^2+x-2}{2x-4} = \frac{4+2-2}{0+} = \frac{4}{0+} = +\infty \quad \tfrac{1}{4}\text{ b}$$

$$\lim_{x\to2^-} \frac{x^2+x-2}{2x-4} = \frac{4}{0-} = -\infty \quad \tfrac{1}{4}\text{ b}$$

f MÁ V BODĚ $x_0 = 2$ SVISLOU ASYMPTOTU

$\frac{1}{4}$ b (6) asymptoty

B

5) PRŮSEČÍKY
celkem $\frac{3}{4}b$

$P_y = [0; \frac{1}{2}]$ $\frac{1}{4}b$

P_x: $\frac{x^2+x-2}{2x-4} = 0 \Leftrightarrow x^2+x-2=0$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \overset{\shortparallel}{(x+2)(x-1)}$

$P_{x_1} = [-2;0] \quad P_{x_2} = [1;0]$ $\frac{1}{2}b$

6) ASYMPTOTY
celkem $2+\frac{1}{4}b$

$\boxed{+\infty}$

$$\lim_{x\to+\infty} \frac{\frac{x^2+x-2}{2x-4}}{x} \overset{\frac{1}{4}b}{=} \lim_{x\to+\infty} \frac{x^2+x-2}{2x^2-4x} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^2}(1+\frac{1}{x}-\frac{2}{x^2})}{\cancel{x^2}(2-\frac{4}{x})} \underset{\text{LIM PŘECH}}{=} \frac{1}{2} \neq 0 \;\; \frac{1}{4}b \Rightarrow$$ f MÁ V $+\infty$ ŠIKMOU ASYMPTOTU $y = \frac{1}{2}x + b$

$$\lim_{x\to+\infty}\left(\frac{x^2+x-2}{2x-4} - \frac{1}{2}x\right) = \lim_{x\to+\infty} \frac{\cancel{x^2}+x-2-\cancel{x^2}+2x}{2x-4} =$$

$$= \lim_{x\to+\infty} \frac{3x-2}{2x-4} = \lim_{x\to+\infty} \frac{\cancel{x}(3-\frac{2}{x})}{\cancel{x}(2-\frac{4}{x})} \underset{\text{LIM PŘECH}}{=} \frac{3}{2} = b \;\; \frac{1}{2}b$$

$\boxed{-\infty}$

$$\lim_{x\to-\infty} \frac{\frac{x^2+x-2}{2x-4}}{x} \overset{\frac{1}{4}b}{=} \lim_{x\to-\infty} \frac{x^2+x-2}{2x^2-4x} =$$

$$= \lim_{x\to-\infty} \frac{\cancel{x^2}(1+\frac{1}{x}-\frac{2}{x^2})}{\cancel{x^2}(2-\frac{4}{x})} \underset{\text{LIM PŘECH}}{=} \frac{1}{2} \neq 0 \;\; \frac{1}{4}b \Rightarrow$$ f MÁ V $-\infty$ ŠIKMOU ASYMPTOTU $y = \frac{1}{2}x + b$

$$\lim_{x\to-\infty}\left(\frac{x^2+x-2}{2x-4} - \frac{1}{2}x\right) = \lim_{x\to-\infty} \frac{3x-2}{2x-4} = \lim_{x\to-\infty} \frac{3-\frac{2}{x}}{2-\frac{4}{x}} =$$

$$\underset{\text{LIM PŘ}}{=} \frac{3}{2} = b \;\; \frac{1}{2}b$$

B

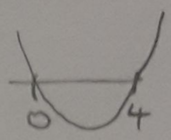
7) $f'(x) = \dfrac{(2x+1)(2x-4) - (x^2+x-2)(2)}{(2x-4)^2} =$

$= \dfrac{4x^2 - 8x \cancel{+2x} \cancel{-4} - 2x^2 \cancel{-2x} \cancel{+4}}{(2x-4)^2} =$

$= \dfrac{2x^2-8x}{(2x-4)^2} = \dfrac{2(x^2-4x)}{4(x-2)^2} = \dfrac{x^2-4x}{2(x-2)^2} =$

$= \dfrac{x(x-4)}{2(x-2)^2} \geq 0$

1b



$D_{f'} = \mathbb{R} \setminus \{2\}$

8)+9) MONOTONIE + LOK EXTRÉMY - 1b celkem

$x \in$	$(-\infty; 0)$	$(0; 2)$	$(2; 4)$	$(4; +\infty)$
$f'(x)$	+	−	−	+
$f(x)$	↗ ROSTE	↘ KLESÁ	↘ KLESÁ	↗ ROSTE

½b

$2 \notin D_f$

$x_0 = 0 \in D_f$

f MÁ V BODĚ $x_0 = 0$ LOK. MAX.

$f(0) = \dfrac{1}{2}$

¼b

$x_0 = 4 \in D_f$

f MÁ V BODĚ $x_0 = 4$ LOK. MIN.

$f(4) = \dfrac{16+4-2}{8-4} =$

$= \dfrac{18}{4} = \dfrac{9}{2} = 4.5$

¼b

10) $f''(x) = \dfrac{(2x-4)(2(x-2)^2) - (x^2-4x)2(x-2)\cdot 2}{4(x-2)^4} =$

$= \dfrac{4(x-2)^3 - 4(x^2-4x)(x-2)}{4(x-2)^4} =$

$= \dfrac{1}{x-2} - \dfrac{x^2-4x}{(x-2)^3} = \dfrac{\cancel{x^2} \cancel{-4x} +4 \cancel{-x^2} \cancel{+4x}}{(x-2)^3} =$

$= \dfrac{4}{(x-2)^3}$, $D_{f''} = \mathbb{R} \setminus \{2\}$

$= \dfrac{4}{(x-2)\underbrace{(x-2)^2}_{\geq 0}}$ $\frac{3}{2}$ b

11) KONVEX/ KONKÁV

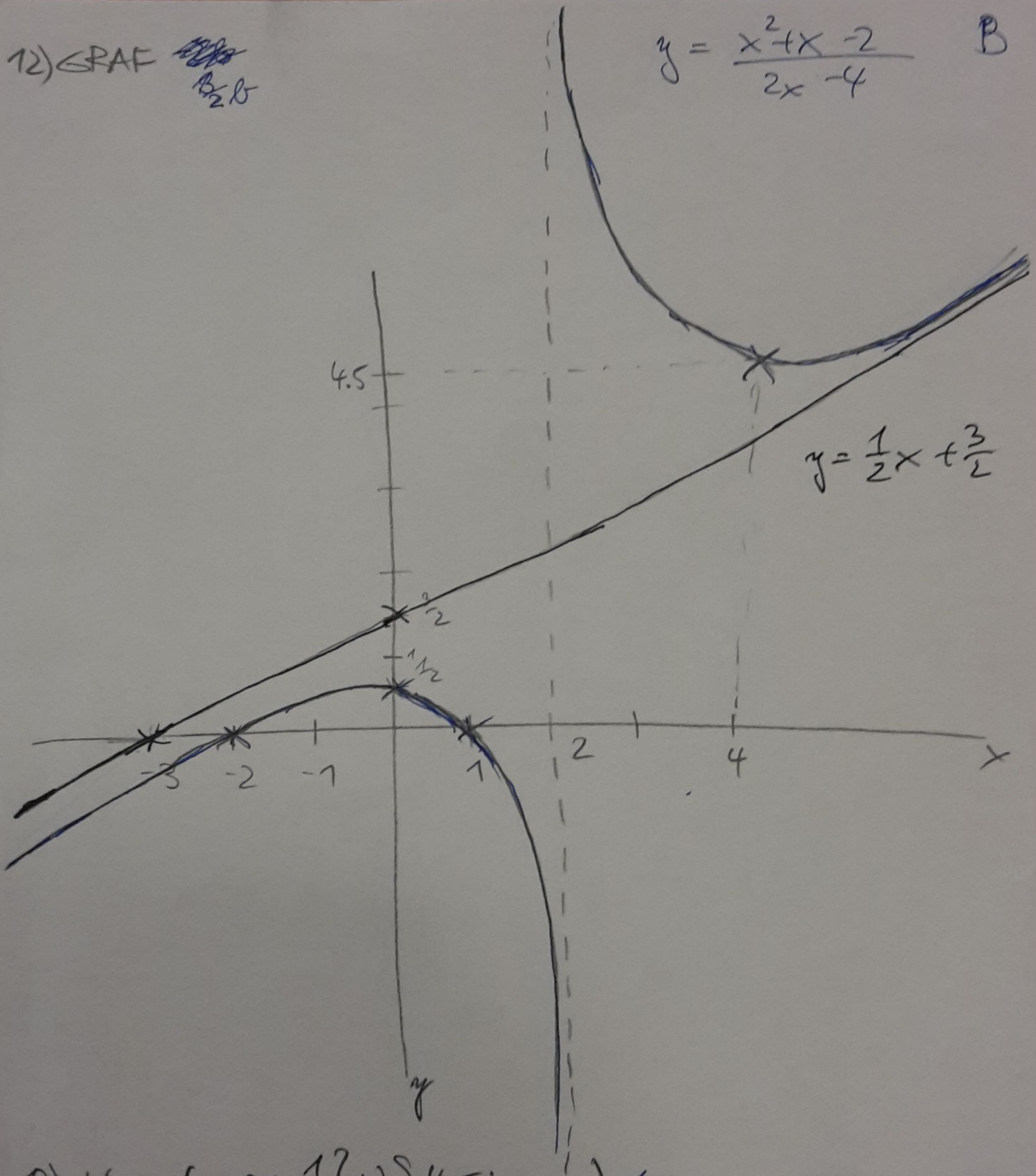
$x \in$	$(-\infty, 2)$	$(2; +\infty)$
$f''(x)$	$-$	$+$
f	$\cap$ konkávní	$\cup$ konvexní

$\frac{1}{2}$ b

$x_0 = 2 \notin D_f \Rightarrow$ NENÍ TO INFLEXNÍ BOD

12) GRAF 3/2 b

$y = \frac{x^2+x-2}{2x-4}$ B



13) $H_f = (-\infty; \frac{1}{2}] \cup [4.5; +\infty)$ 1/4 b

14) f NEMÁ GLOBÁLNÍ EXTRÉMY 1/4 b