A1 PT LS 2018/2019

$$\lim \frac{\left(\frac{1}{2}\right)^{n+1} + \left(\frac{2}{3}\right)^{2n}}{\left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{3}\right)^{2n}} = \lim \frac{\frac{1}{2}\cdot\left(\frac{1}{2}\right)^{n} + \left(\frac{4}{9}\right)^{n}}{2\cdot\left(\frac{1}{2}\right)^{n} + \left(\frac{1}{9}\right)^{n}} =$$

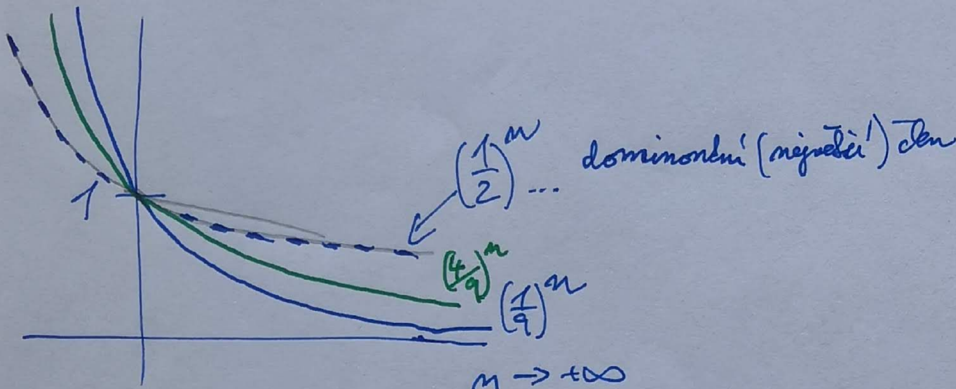
ÚPRAVA 1b

$$= \lim \frac{\cancel{\left(\frac{1}{2}\right)^{n}} \left[\frac{1}{2} + \left(\frac{8}{9}\right)^{n}\right]}{\cancel{\left(\frac{1}{2}\right)^{n}} \left[2 + \left(\frac{2}{9}\right)^{n}\right]} = \frac{\frac{1}{2} + 0}{2 + 0} = \frac{1}{4}$$

VYTKNUTÍ 1b

DOPOČTENÍ 1b

$\boxed{3b}$



A2

$f(x) = \frac{1}{2}x^2 + x - 4$

TEČNA:

$f'(x) = \boxed{x + 1 = 2}$ 0.4b — chceme, aby smernice bola 2

$x_0 = 1$

$f(1) = \frac{1}{2} + 1 - 4 = -\frac{5}{2}$

$y = 2(x-1) - \frac{5}{2} = 2x - \frac{9}{2}$ 1b

Průsečíky tečny s osami: $P_y^T = [0; -\frac{9}{2}]$ $P_x^T = [\frac{9}{4}; 0]$ 0.4b

PARABOLA:

$P_y = [0; -4]$ 0.2b

$P_x$: $\frac{1}{2}x^2 + x - 4 = 0$

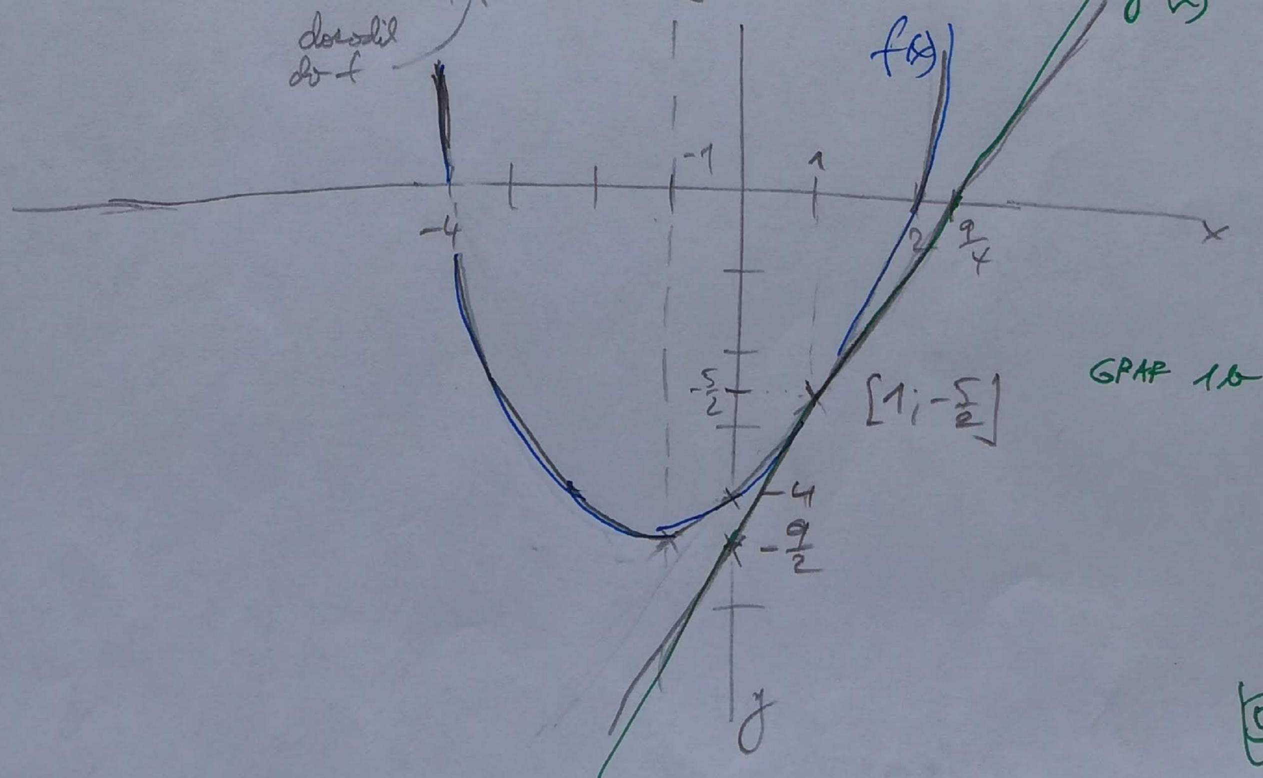
$x^2 + 2 - 8 = 0$ $\longrightarrow$ $P_{x_1} = [-4; 0]$ $P_{x_2} = [2; 0]$ 1b

$x_1 = -4$

$x_2 = 2$

$V = [\frac{x_1 + x_2}{2}; ?] = [-1; \frac{1}{2} - 1 - 4] = [-1; -\frac{9}{2}]$ 1b

dosadil do f




GRAF 1b

5b

A3

$$f(x)=\frac{x^2-x-6}{4-2x}=\frac{(x+2)(x-3)}{-2(x-2)}$$

1) $D_f$ : $4-2x=0$ $\quad D_f=(-\infty;2)\cup(2;+\infty)$ $\frac{1}{4}$b

2) S/L $f(1)=\frac{1-1-6}{4-2}=-3$ $\quad f(1)\neq f(-1) \Rightarrow f$ NENÍ SUDÁ

$f(-1)=\frac{1+1-6}{4+2}=-\frac{4}{6}=-\frac{2}{3}$ $\quad f(1)\neq -f(-1) \Rightarrow f$ NENÍ LICHÁ

$\frac{1}{2}$b

ALTERNATIVNĚ:
$D_f$ "NESYMETRICKÝ"
$\Rightarrow f$ NENÍ S/L

3) $P_y=[0;-\frac{3}{2}]$ $\frac{1}{4}$b

$P_x$: $\frac{x^2-x-6}{4-2x}=0 \Leftrightarrow x^2-x-6=0$

$x_1=-2$ $\quad P_{x_1}=[-2;0]$

$x_2=3$ $\quad P_{x_2}=[3;0]$ $\quad \frac{1}{2}$b

4) +/−

$\frac{1}{2}$b

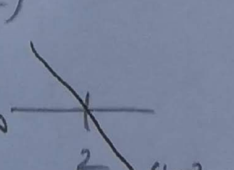
| | $(-\infty;-2)$ | $(-2;2)$ | $(2;3)$ | $(3;+\infty)$ |
|---|---|---|---|---|
| $(x+2)$ | − | + | + | + |
| $(x-2)$ | − | − | + | + |
| $(x-3)$ | − | − | − | + |
| $-2$ | − | − | − | − |
| $f(x)$ | + | − | + | − |

5)

$$\lim_{x\to+\infty}\frac{x^2-x-6}{4-2x}=\lim_{x\to+\infty}\frac{x^{\cancel{2}}(1-\frac{1}{x}-\frac{6}{x^2})}{\cancel{x}(\frac{4}{x}-2)}=\frac{+\infty(1-0-0)}{0-2}=-\infty$$ $\frac{1}{4}$b

$$\lim_{x\to-\infty}\frac{x^2-x-6}{4-2x}=\lim_{x\to-\infty}\frac{x^{\cancel{2}}(1-\frac{1}{x}-\frac{6}{x^2})}{\cancel{x}(\frac{4}{x}-2)}=\frac{-\infty(1-0-0)}{0-2}=+\infty$$ $\frac{1}{4}$b

$$\lim_{x\to2^+}\frac{x^2-x-6}{4-2x}=\frac{4-2-6}{0-}=\frac{-4}{0-}=+\infty$$ $\frac{1}{4}$b




$$\lim_{x\to2^-}\frac{x^2-x-6}{4-2x}=\frac{-4}{0+}=-\infty$$ $\frac{1}{4}$b

} $f$ MÁ V 2 SVISLOU ASYMPTOTU $\frac{1}{4}$b

6)

Asymptota v $+\infty$

$$\lim_{x\to+\infty}\frac{f(x)}{x}=\lim_{x\to+\infty}\frac{\frac{x^2-x-6}{4-2x}}{x}=\lim_{x\to+\infty}\frac{x^2-x-6}{4x-2x^2}=$$

$$=\lim_{x\to+\infty}\frac{\cancel{x^2}(1-\frac{1}{x}-\frac{6}{x^2})}{\cancel{x^2}(\frac{4}{x}-2)}=\frac{1-0-0}{0-2}=\underbrace{-\frac{1}{2}}_{a=}$$

f MÁ V $+\infty$ ŠIKMOU ASYMPTOTU $y=-\frac{1}{2}x+b$ $\frac{1}{2}b$

$$\lim_{x\to+\infty} f(x)-ax=\lim_{x\to+\infty}\frac{x^2-x-6}{4-2x}+\frac{1}{2}x=\lim_{x\to+\infty}\frac{\cancel{x^2}-x-6+2x-\cancel{x^2}}{4-2x}=$$

$$=\lim_{x\to+\infty}\frac{x-6}{4-2x}=\lim_{x\to+\infty}\frac{\cancel{x}(1-\frac{6}{x})}{\cancel{x}(\frac{4}{x}-2)}=\frac{1-0}{0-2}=-\frac{1}{2}$$

$\frac{1}{3}b$

v $-\infty$ ... STEJNÝ POSTUP.. f MÁ V $-\infty$ ŠIKMOU ASYMPTOTU $y=-\frac{1}{2}x-\frac{1}{2}$ 1b

7) $$f'(x)=\frac{(2x-1)(4-2x)-(x^2-x-6)(-2)}{(4-2x)^2}=\frac{8x-4x^2-4+\cancel{2x}+2x^2-\cancel{2x}-12}{(4-2x)^2}=$$

$$=\frac{-2x^2+8x-16}{(4-2x)^2}=\frac{-2(x^2-4x+8)}{(4-2x)^2}=\frac{x^2-4x+8}{-2(2-x)^2}=$$

$$=\frac{\overbrace{(x^2-4x+8)}^{>0}}{\underbrace{-2}_{<0}\underbrace{(x-2)^2}_{>0}}$$ 1,5b

8) $f'$ +/-

$x^2-4x+8=0$

$D=16-4\cdot 8=-16<0 \longrightarrow x^2-4x+8>0$

$\frac{3}{4}b$

$f'(x)<0\quad \forall x\in D_f$ ... f JE KLESAJÍCÍ NA $(-\infty;2)$ A $(2;+\infty)$

9) $$f''(x) = \frac{\cancel{[2x-4]}^{-2}\left(\cancel{2}(x-2)^2\right) - \left[x^2-4x+8\right]\cancel{(4)}(x-2)}{\cancel{4}(x-2)^4} =$$

$$= \frac{-(x-2)^2 + x^2 - 4x + 8}{(x-2)^3} = \frac{\cancel{-x^2} + \cancel{4x} - 4 \cancel{+x^2} - \cancel{4x} + 8}{(x-2)^3} =$$

$$= \frac{4}{(x-2)^3} = \frac{4}{\underbrace{(x-2)^2}_{>0}(x-2)} \quad \frac{3}{2}b.$$

10) $f''$ +/−

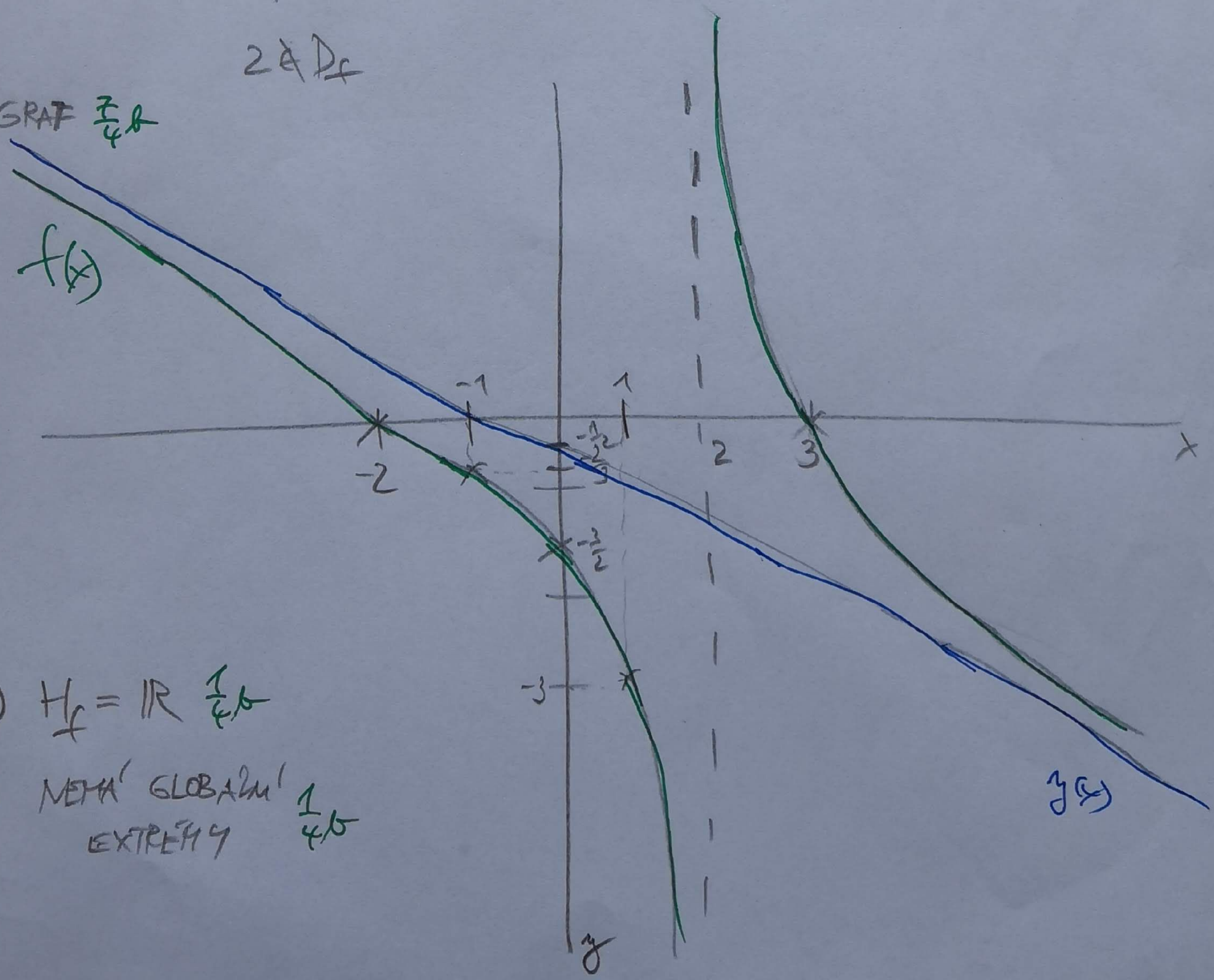
| | $(-\infty; 2)$ | $(2; +\infty)$ |
|---|---|---|
| $(x-2)$ | − | + |
| $f''(x)$ | − | + |
| $f(x)$ | ∩ KONKÁVNÍ | ∪ KONVEXNÍ |

$2 \notin D_f$

f NEMÁ INFLEXNÍ BOD

$\frac{3}{4}b$

11) GRAF $\frac{7}{4}b$




12) $H_f = \mathbb{R}$ $\frac{1}{4}b$

13) NEMÁ GLOBÁLNÍ EXTRÉMY $\frac{1}{4}b$