B1 PT LS 2018/2019

$$\lim \frac{\left(\frac{1}{2}\right)^{n-1}+\left(\frac{1}{3}\right)^{2n}}{\left(\frac{1}{2}\right)^{n+1}+\left(\frac{2}{3}\right)^{2n}} = \lim \frac{2\left(\frac{1}{2}\right)^{n}+\left(\frac{1}{9}\right)^{n}}{\frac{1}{2}\left(\frac{1}{2}\right)^{n}+\left(\frac{4}{9}\right)^{n}} =$$

ÚPRAVA 1b

$$= \lim \frac{\cancel{\left(\frac{1}{2}\right)^{n}}\left[2+\left(\frac{2}{9}\right)^{n}\right]}{\cancel{\left(\frac{1}{2}\right)^{n}}\left[\frac{1}{2}+\left(\frac{8}{9}\right)^{n}\right]} = \frac{2+0}{\frac{1}{2}+0} = 4$$

VYTKNUTÍ 1b

DOPOČTENÍ 1b

$\boxed{3b}$

B2

$$f(x) = \frac{1}{2}x^2 - x - 4$$

TEČNA:

$$f'(x) = \boxed{x-1=-2}$$ chceme, aby byla směrnice -2 0.4b

$$x_0 = -1$$

$$f(-1) = \frac{1}{2}+1-4 = -\frac{5}{2}$$

$$y = -2(x+1) - \frac{5}{2} = -2x - \frac{9}{2}$$ 0.4b

Průsečíky tečny s osami: $P_y^T = \left[0; -\frac{9}{2}\right]$; $P_x^T = \left[-\frac{9}{4}; 0\right]$ 0.4b

PARABOLA:

$P_y = [0; -4]$ 0.2b

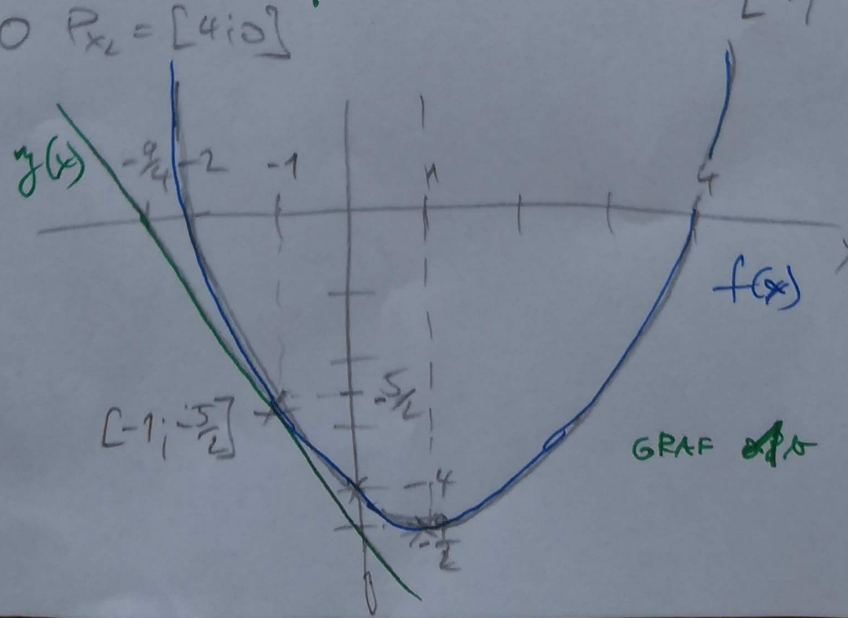
$$V = \left[\frac{x_1+x_2}{2}; ?\right] = \left[1; \frac{1}{2}-1-4\right] = \left[1; -\frac{9}{2}\right]$$ 0.4b

P_x: $\frac{1}{2}x^2 - x - 4 = 0$ $P_{x_1} = [-2; 0]$ 0.4b

$x^2 - 2x - 8 = 0$ $P_{x_2} = [4; 0]$

$x_1 = -2$

$x_2 = 4$



GRAF 0.4b

$\boxed{5b}$

B3

$$f(x)=\frac{-x^2-x+6}{2x+4}=\frac{-(x+3)(x-2)}{2(x+2)}$$

1) $D_f:\ 2x+4=0 \qquad D_f=(-\infty;-2)\cup(-2;+\infty)$ $\frac{1}{4}b$

2) $f(1)=\frac{-1-1+6}{2+4}=\frac{4}{6}=\frac{2}{3}$

$f(-1)=\frac{-1+1+6}{-2+4}=\frac{6}{2}=3$

$f(1)\neq f(-1) \Rightarrow$ f NENÍ SUDÁ

$f(1)\neq -f(-1) \Rightarrow$ f NENÍ LICHÁ $\frac{1}{2}b$

3) $P_y=[0;\frac{3}{2}]$ $\frac{1}{4}b$

$P_x:\ \frac{-x^2-x+6}{2x+4}=0 \Leftrightarrow -x^2-x+6=0$

$x^2+x-6=0$

$x_1=-3$

$x_2=2$

$P_{x_1}=[-3;0]$

$P_{x_2}=[2;0]$ $\frac{1}{2}b$

4) +/−

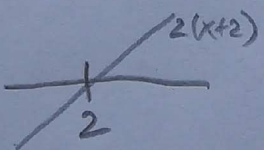
	$(-\infty;-3)$	$(-3;-2)$	$(-2;2)$	$(2;+\infty)$
$(x+3)$	−	+	+	+
$(x+2)$	−	−	+	+
$(x-2)$	−	−	−	+
−	−	−	−	−
$f(x)$	+	−	+	−

$\frac{1}{2}b$

5) $\lim\limits_{x\to+\infty}\frac{-x^2-x+6}{2x+4}=\lim\limits_{x\to+\infty}\frac{x^{\cancel{2}}(-1-\frac{1}{x}+\frac{6}{x^2})}{\cancel{x}(2+\frac{4}{x})}=\frac{+\infty(-1-0+0)}{2+0}=-\infty$ $\frac{1}{4}b$

$\lim\limits_{x\to-\infty}\frac{-x^2-x+6}{2x+4}=\lim\limits_{x\to-\infty}\frac{x(-1-\frac{1}{x}+\frac{6}{x^2})}{2+\frac{4}{x}}=\frac{-\infty(-1-0+0)}{2+0}=+\infty$ $\frac{1}{4}b$

$\lim\limits_{x\to-2^+}\frac{-x^2-x+6}{2x+4}=\frac{-4+2+6}{0_+}=\frac{+4}{0_+}=+\infty$ $\frac{1}{4}b$



f MÁ V −2 SVISLOU ASYMPTOTU $\frac{1}{4}b$

$\lim\limits_{x\to-2^-}\frac{-x^2-x+6}{2x+4}=\frac{+4}{0_-}=-\infty$ $\frac{1}{4}b$

6)

Asymptota v $+\infty$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\frac{-x^2-x+6}{2x+4}}{x} = \lim_{x\to+\infty} \frac{-x^2-x+6}{2x^2+x} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^2}\left(-1-\frac{1}{x}+\frac{6}{x}\right)}{\cancel{x^2}\left(2+\frac{1}{x}\right)} = \frac{-1-0+0}{2+0} = \underbrace{-\frac{1}{2}}_{a=}$$ $\frac{1}{2}b$

f MÁ V $+\infty$ ŠIKMOU ASYMPTOTU $y=-\frac{1}{2}x+b$

$$\lim_{x\to+\infty} f(x)-ax = \lim_{x\to+\infty} \frac{-x^2-x+6}{2x+4} + \frac{1}{2}x =$$

$$= \lim_{x\to+\infty} \frac{\cancel{-x^2}-x+6+\cancel{x^2}+2x}{2x+4} = \lim_{x\to+\infty} \frac{x+6}{2x+4} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\left(1+\frac{6}{x}\right)}{\cancel{x}\left(2+\frac{4}{x}\right)} = \frac{1+0}{2+0} = \frac{1}{2} = b$$ $\frac{1}{2}b$

Asymptota v $-\infty$ ... STEJNÝ POSTUP f MÁ V $-\infty$ ŠIKMOU ASYMPTOTU $y=-\frac{1}{2}x+\frac{1}{2}$ 1b

7)
$$f'(x) = \frac{(-2x-1)(2x+4)-(-x^2-x+6)(2)}{4(x+2)^2} =$$

$$= \frac{-4x^2-8x\cancel{-2x}-4+2x^2\cancel{+2x}-12}{4(x+2)^2} =$$

$$= \frac{-2x^2-8x-16}{4(x+2)^2} = \frac{(x^2+4x+8)}{\underbrace{-2}_{<0}\underbrace{(x+2)^2}_{>0}} > 0$$ $\frac{3}{2}b$

8) f' +/−

$x^2+4x+8=0$

$D = 16-4\cdot 8 = -16 < 0 \rightarrow x^2+4x+8>0$

$f'(x)<0 \quad \forall x \in D_f \Rightarrow f$ JE KLESAJÍCÍ NA $(-\infty;2)$ A $(2;+\infty)$ $\frac{3}{4}b$

9) $f''(x) = \dfrac{(\cancel{2x}\overset{+2}{\cancel{+4}})(-2(x+2)^2) - (x^2+4x+8)\cancel{(-4)}(x+2)(1)}{\cancel{4}(x+2)^4} =$

$= \dfrac{-(x+2)^2 \; + x^2+4x+8}{(x+2)^3} = \dfrac{\cancel{-x^2}\cancel{-4x}-4 \cancel{+x^2}\cancel{+4x}+8}{(x+2)^3} =$

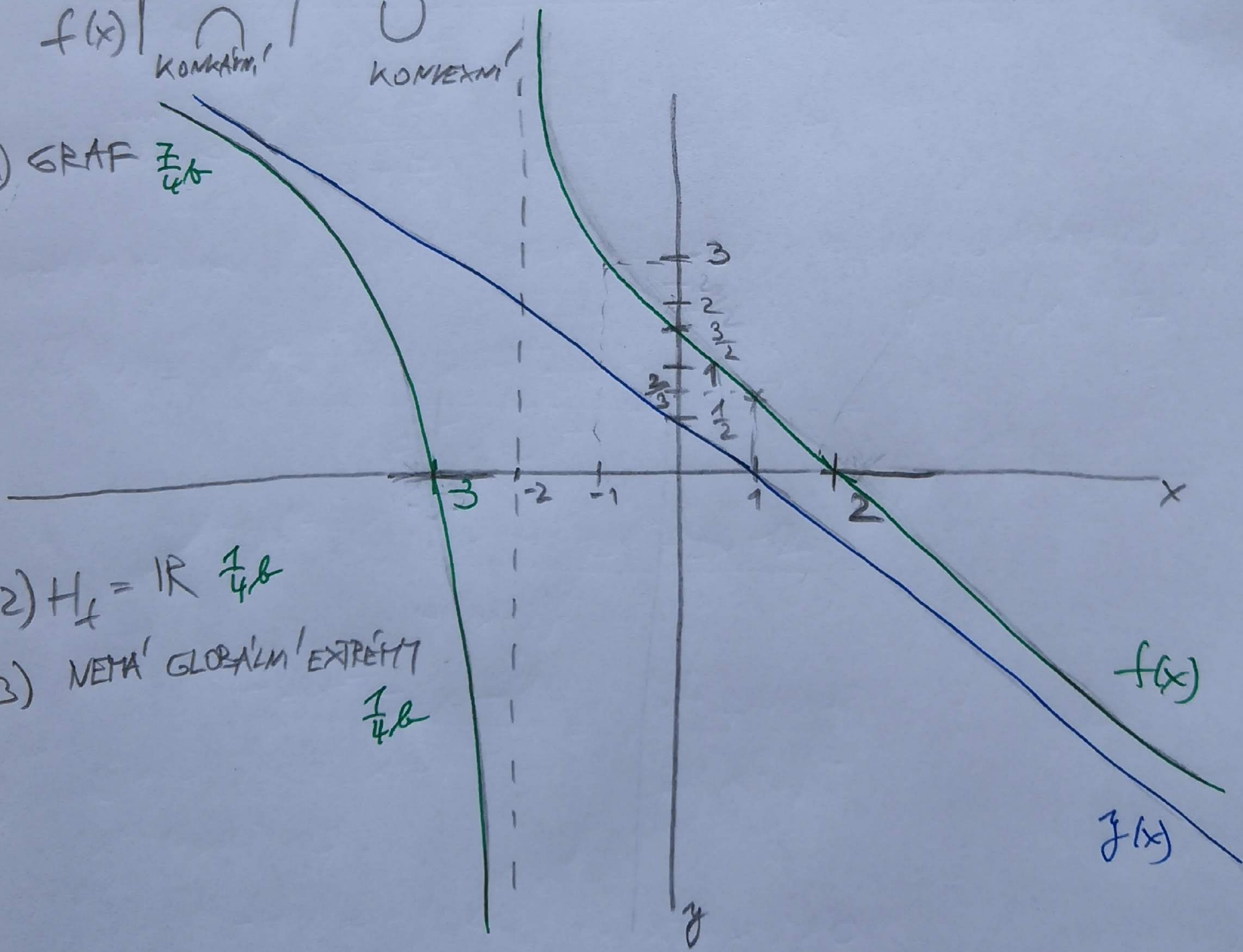
$= \dfrac{4}{(x+2)^3} = \dfrac{4}{\underbrace{(x+2)^2}_{>0}(x+2)}$ $\frac{3}{2}$ b

10) f'' +/−

	$(-\infty; -2)$	$(-2; +\infty)$
$(x+2)$	−	+
$f''(x)$	⊖	⊕
$f(x)$	∩ KONKÁVNÍ	∪ KONVEXNÍ

f NEMÁ INFLEXNÍ BOD $\frac{3}{4}$ b

11) GRAF $\frac{1}{4}$ b



12) $H_f = \mathbb{R}$ $\frac{1}{4}$ b

13) NEMÁ GLOBÁLNÍ EXTRÉMY $\frac{1}{4}$ b