PT ZS 17/18 VAR D

1.

$$\lim \frac{2^{2n}-3^{n+1}}{5\cdot 3^n+4^{n+1}} = \lim \frac{4^n - 3\cdot 3^n}{5\cdot 3^n + 4\cdot 4^n} =$$ 1b úprava

$$= \lim \frac{4^n\left(1-3\cdot\left(\frac{3}{4}\right)^n\right)}{4^n\left(5\cdot\left(\frac{3}{4}\right)^n+4\right)} = \frac{1-3\cdot 0}{5\cdot 0+4} = \frac{1}{4}$$ 1b dosazení

1 bod vytknutí dom. člene

3b

2. $f(x) = \frac{1}{2}x^2 - x - 4$, $x_0 = 3$

TEČNA

$f'(x) = x - 1$ $\frac{1}{2}$b

$f'(3) = 3 - 1 = 2$

$f(3) = \frac{1}{2}9 - 3 - 4 = \frac{9-6-8}{2} = -\frac{5}{2}$

TEČNA V BODĚ $x_0 = 3$: $y = 2(x-3) - \frac{5}{2} =$

$= 2x - 6 - \frac{5}{2} =$

$= 2x - \frac{17}{2}$ 1b

Průsečíky tečny s osami x, y: $P_y^T = [0; -\frac{17}{2}]$ $P_x^T = [\frac{17}{4}; 0]$ $\frac{1}{4}$b

PARABOLA

$P_y = [0; -4]$ $\frac{1}{4}$b

P_x: $\frac{1}{2}x^2 - x - 4 = 0$

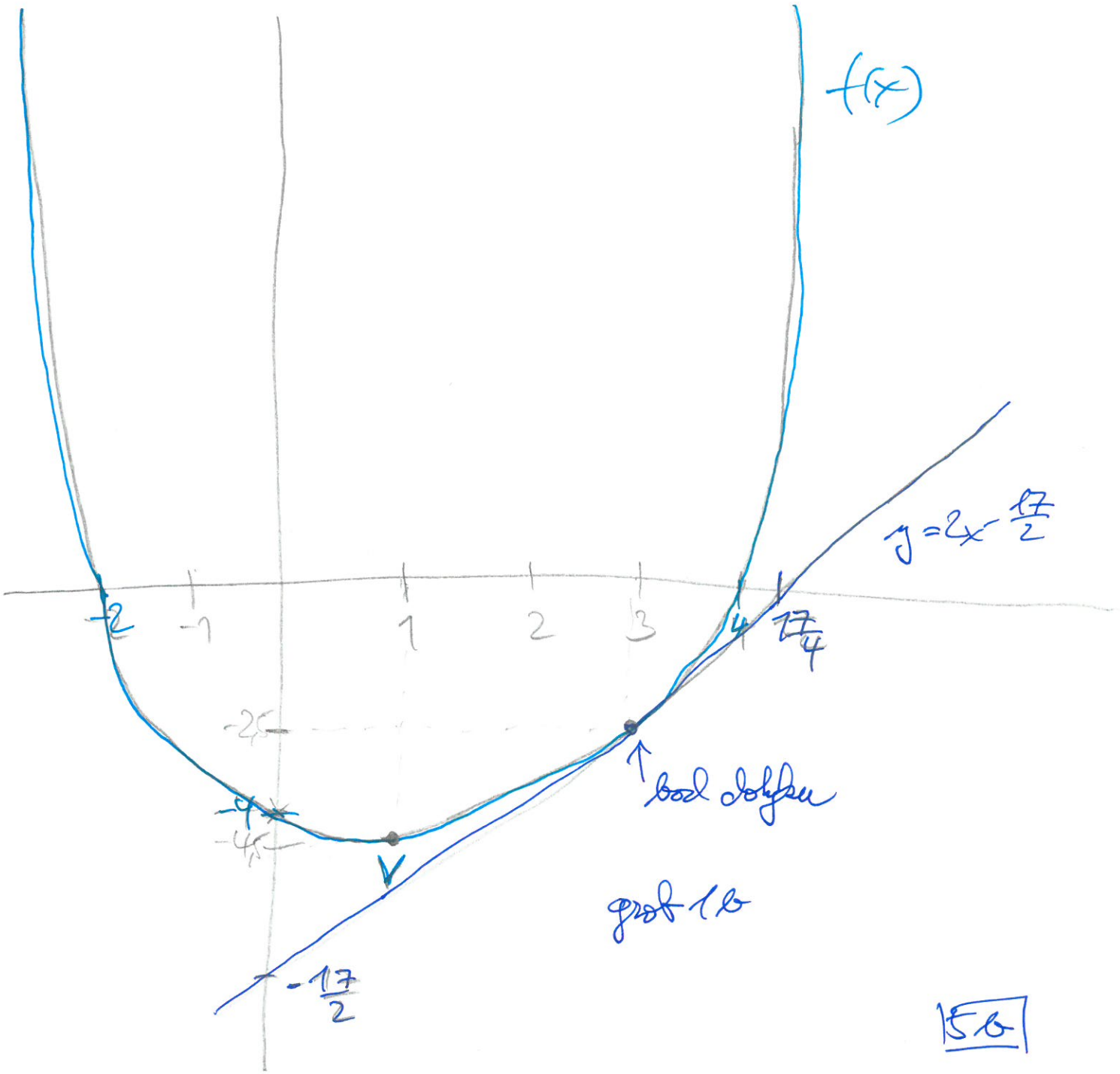
$x^2 - 2x - 8 = 0$

$x_1 = 4 \Rightarrow P_{x_1} = [4; 0], P_{x_2} = [-2; 0]$ kořeny 1b

$x_2 = -2$

$V = [\frac{x_1+x_2}{2}; ?] = [\frac{4-2}{2}; ?] = [1; -\frac{9}{2}]$ vrchol 1b

$f(1) = \frac{1}{2} - 1 - 4 = -\frac{9}{2}$



5b

3. $f(x) = (x+1)\,e^{1-x}$

1) $D_f = \mathbb{R}$ $\frac{1}{4}$b

sudost/lichost $\frac{1}{4}$b

$f(-1) = 0$ $\quad f(-1) \neq f(1) \Rightarrow f$ není sudá

$f(1) = 2e^{1-1} = 2$ $\quad f(-1) \neq -f(1) \Rightarrow f$ není lichá

2) limity v krajních bodech D_f celkem $\frac{3}{2}$b

$$\lim_{x\to+\infty} (x+1)e^{1-x} = \lim_{x\to+\infty} \frac{x+1}{e^{(-1+x)}} = \lim_{x\to+\infty} \frac{1}{e^{-1+x}\cdot 1} =$$

„$\infty\cdot 0$" $\quad$ „$\frac{\infty}{\infty}$" L'H

různá $\frac{1}{2}$b $\quad$ součin L'H $\frac{1}{2}$b

$$= \lim_{x\to+\infty} e^{1-x} = 0$$ dopočtení $\frac{1}{4}$b

$$\lim_{x\to-\infty} (x+1)e^{1-x} = (-\infty+1)(+\infty) = (-\infty)(+\infty) = -\infty$$ $\frac{1}{4}$b

3) Průsečíky celkem $\frac{3}{4}$b

$P_y = [0; e]$ $\frac{1}{4}$b

P_x: $(x+1)\underbrace{e^{1-x}}_{>0} = 0$

$x+1 = 0$

$x = -1 \Rightarrow P_x = [-1; 0]$ $\frac{1}{2}$b

4) Asymptoty celkem $\frac{5}{4}$b

$0+\infty$

z2) víme $\lim_{x\to+\infty} f(x) = 0 \Rightarrow f$ má v $+\infty$ asymptotu $y = 0$

$\frac{1}{2}$b

$-\infty$

$$\lim_{x\to-\infty}\frac{f(x)}{x}=\lim_{x\to-\infty}\frac{(x+1)e^{1-x}}{x}=$$

$\frac{1}{4}b$

$$=\left(\lim_{x\to-\infty}\frac{x+1}{x}\right)\cdot\left(\lim_{x\to-\infty}e^{1-x}\right)=1\cdot(+\infty)=+\infty$$

dokončení $\frac{1}{2}b$

$\Rightarrow$ f nemá $-\infty$ asymptotu

5) $f'(x)=1\cdot e^{1-x}+(x+1)e^{1-x}(-1)=$

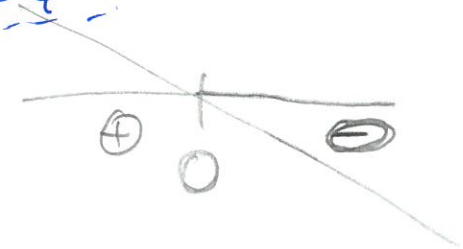
$=e^{1-x}(1-x-1)=(-x)e^{1-x}$

>0

$\frac{7}{4}b$

$D_{f'}=\mathbb{R}$

6)+7) monotonie + lok. extrémy (alebo $\frac{3}{4}b$)



$(-\infty;0)$	$(0;+\infty)$
$f'(x)>0$	$f'(x)<0$
f ROSTE	KLESÁ

$\frac{1}{2}b$

$\Downarrow$

$0\in D_f$
f má v bodě 0
lokální maximum
$f(0)=e$

$\frac{1}{4}b$

8) $f''(x)=-1\cdot e^{1-x}+(-x)e^{1-x}\cdot(-1)=$

$=(x-1)e^{1-x}$

>0

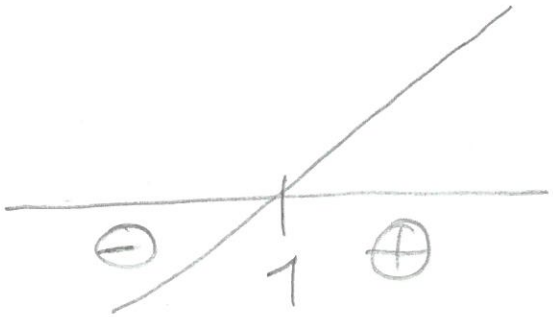
$\frac{7}{4}b$

$D_{f''}=\mathbb{R}$

9) konvex/konkáv celkom $\frac{3}{4}$b

$(-\infty; 1)$	$(1; +\infty)$
$f''(x) < 0$	$f''(x) > 0$
f je KONKÁVNA	KONVEXNÁ

$\frac{1}{2}$b

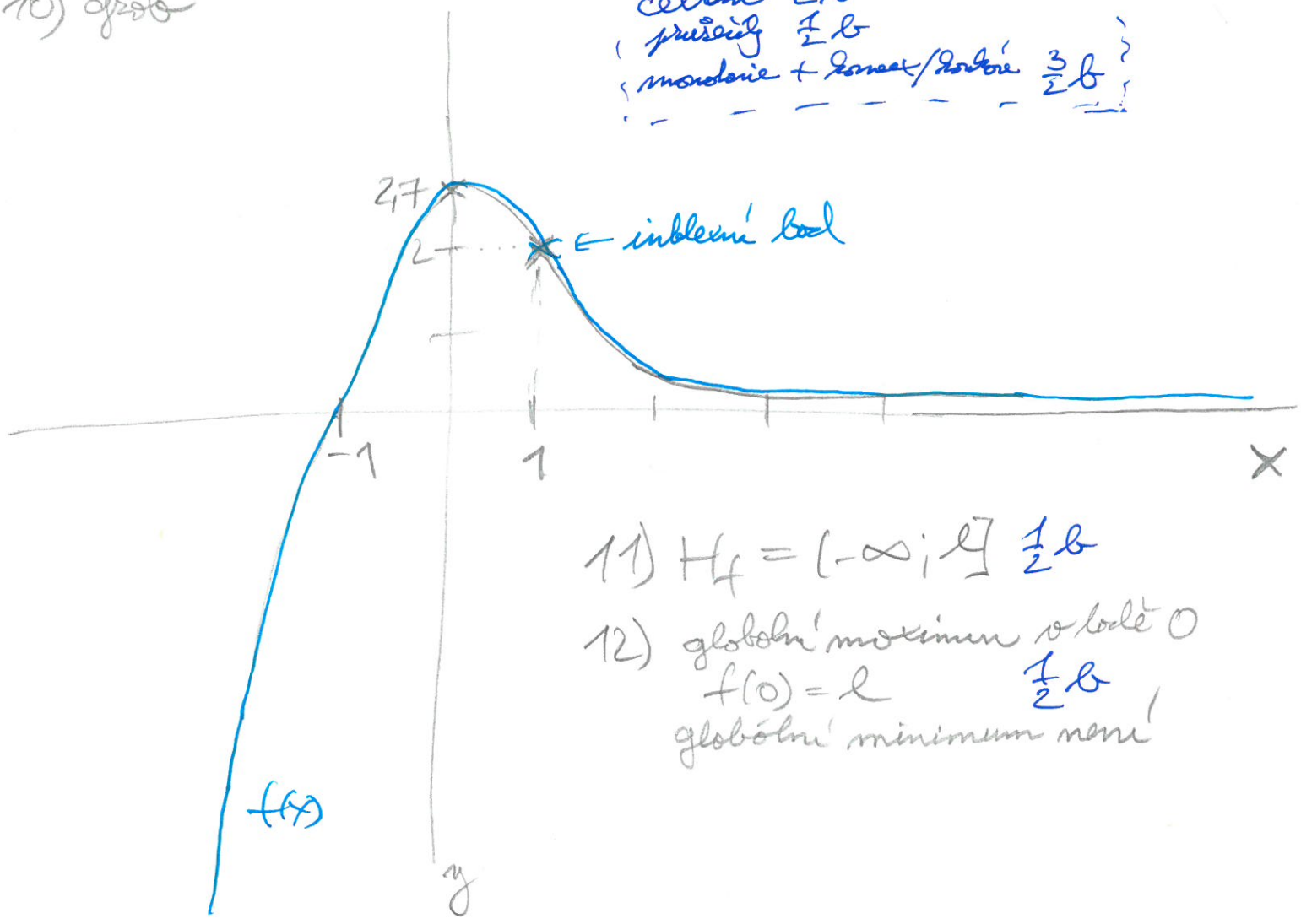


$1 \in D_f$
f má v bode 1 inflexný bod
$f(1) = 2 \quad \ell^0 = 2$

$\frac{1}{4}$b

10) graf

celkom 2b
priesečky $\frac{1}{2}$b
monotónie + konvex/konkáv $\frac{3}{2}$b



11) $H_f = (-\infty; 2]$ $\frac{1}{2}$b

12) globálne maximum v bode 0
$f(0) = 2$ $\frac{1}{2}$b
globálne minimum nemá