Určete všechny asymptoty v $\pm\infty$ i svislé. Nakreslete asymptoty – vrcholy, svislé čáry.

$D_f = ?$

$\boxed{\text{MINITEST CV11}}$

A: $f(x) = \dfrac{-x^3+4x^2-7x+9}{x^2+7x+6}$

D_f: $x^2+7x+6 \neq 0$

$\cancel{D = 49 - 4\cdot 6 = 25}$ $x_1 = -6$, $x_2 = -1$ … svislé asymptoty

$\cancel{x_{1,2}}$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{+x^{\cancel{3}}\left(-1+\frac{4}{x}-\frac{7}{x^2}+\frac{9}{x^3}\right)}{\cancel{x^2}\left(1+\frac{7}{x}+\frac{6}{x^2}\right)} =$$

$$= \left(\lim_{x\to+\infty} x\right)\left(\lim_{x\to+\infty} -1+\frac{4}{x}-\frac{7}{x^2}+\frac{9}{x^3}\right) = -\infty$$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\left(-1+\frac{4}{x}-\frac{7}{x^2}+\frac{9}{x^3}\right)}{1+\frac{7}{x}+\frac{6}{x^2}} = -1 = a$$

$$\lim_{x\to+\infty} f(x)+x = \lim_{x\to+\infty} \frac{-x^3+4x^2-7x+9 \quad +x^3+7x^2+6x}{x^2+7x+6} =$$

$$= \lim_{x\to+\infty} \frac{11x^2-x+9}{x^2+7x+6} = 11$$

Asymptota v $+\infty$ je $y = -x + 11$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{-x^3+4x^2-7x+9}{x^2+7x+6} =$$

$$= \left(\lim_{x\to-\infty} x\right)\left(\lim_{x\to-\infty} \frac{-1+\frac{4}{x}-\frac{7}{x^2}+\frac{9}{x^3}}{1+\frac{7}{x}+\frac{6}{x^2}}\right) = -\infty \cdot -1 =$$

$$= +\infty$$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\left(-1+\frac{4}{x}-\frac{7}{x^2}+\frac{9}{x^3}\right)}{1+\frac{7}{x}+\frac{6}{x^2}} = -1 = a$$

$$\lim_{x\to-\infty} f(x)+x = \lim_{x\to-\infty} \frac{-x^3+4x^2-7x+9 + x^3+7x^2+6x}{x^2+7x+6} =$$

$$= \lim_{x\to-\infty} \frac{11x^2-x+9}{x^2+7x+6} = 11$$

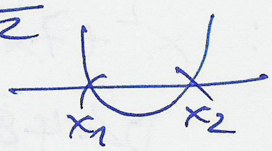
Asymptota v $-\infty$ je $y = -x+11$

B: $f(x)=\sqrt{4x^2+8x+1}$

D_f: $4x^2+8x+1 \overset{?}{\geq} 0$

$D = 64-4\cdot 4\cdot 1 = 64-16 = 48$

$x_{1,2} = \frac{-8\pm\sqrt{48}}{8} = \frac{-8\pm\sqrt{16\cdot 3}}{8} = -1 \pm \frac{\sqrt{3}}{2}$



$x_1 = -1-\frac{\sqrt{3}}{2}$ $\quad D_f = \mathbb{R}\setminus(x_1; x_2)$

$x_2 = -1+\frac{\sqrt{3}}{2}$ $\quad \Rightarrow$ nemá svislé asymptoty

$$\lim_{x\to+\infty}\sqrt{4x^2+8x+1} = \lim_{x\to+\infty} x\cdot\sqrt{4+\frac{8}{x}+\frac{1}{x^2}} = +\infty\cdot 2 = +\infty$$

$$\lim_{x\to+\infty}\frac{f(x)}{x} = \lim_{x\to+\infty}\frac{x\sqrt{4+\frac{8}{x}+\frac{1}{x^2}}}{x} = 2 = a$$

$$\lim_{x\to+\infty} f(x)-2x = \lim_{x\to+\infty}\left(\sqrt{4x^2+8x+1}-\sqrt{4x^2}\right)\cdot\frac{\sqrt{4x^2+8x+1}+\sqrt{4x^2}}{\sqrt{4x^2+8x+1}+\sqrt{4x^2}} =$$

$$= \lim_{x\to+\infty}\frac{8x+1}{\sqrt{4x^2+8x+1}+\sqrt{4x^2}} = \lim_{x\to+\infty}\frac{\cancel{x}\left(8+\frac{1}{x}\right)}{\cancel{x}\sqrt{4+\frac{8}{x}+\frac{1}{x^2}}+\cancel{x}\sqrt{4}} =$$

$$= \frac{8+0}{\sqrt{4}+\sqrt{4}} = \frac{8}{2+2} = \frac{8}{4} = 2$$

Asymptota v $+\infty$ je $y = 2x+2$.

$$\lim_{x\to-\infty}\sqrt{4x^2+8x+1}=\lim_{x\to-\infty}(-x)\sqrt{4+\frac{8}{x}+\frac{1}{x^2}}=+\infty\cdot\sqrt{4}=+\infty$$

$$\lim_{x\to-\infty}\frac{\sqrt{4x^2+8x+1}}{x}=\lim_{x\to-\infty}\frac{(-x)\sqrt{4+\frac{8}{x}+\frac{1}{x^2}}}{x}=(-1)\cdot\sqrt{4}=$$
$$=-2=a$$

$$\lim_{x\to-\infty}\sqrt{4x^2+8x+1}-(-2x)=\lim_{x\to-\infty}\sqrt{4x^2+8x+1}-\sqrt{4x^2}=$$

$$=\lim_{x\to-\infty}\left(\sqrt{4x^2+8x+1}-\sqrt{4x^2}\right)\frac{\sqrt{4x^2+8x+1}+\sqrt{4x^2}}{\sqrt{4x^2+8x+1}+\sqrt{4x^2}}=$$

$$=\lim_{x\to-\infty}\frac{8x+1}{\sqrt{4x^2+8x+1}+\sqrt{4x^2}}=\lim_{x\to-\infty}\frac{\cancel{x}(8+\frac{1}{x})}{-\cancel{x}\sqrt{4+\ldots}+(-\cancel{x})\sqrt{4}}=$$

$$=\frac{8+0}{-\sqrt{4}-\sqrt{4}}=\frac{8}{-4}=-2$$

Asymptota v $-\infty$ je $y=-2x-2$.