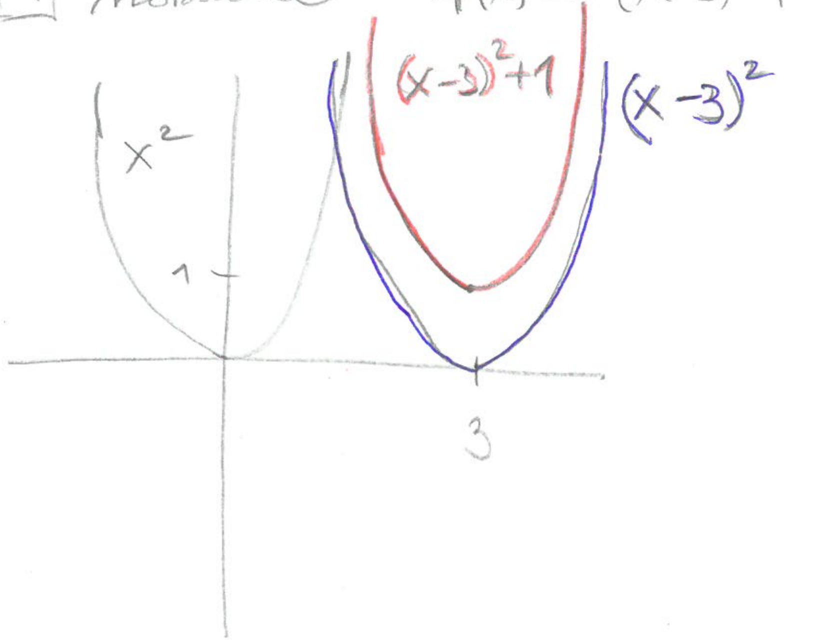
I. Kvadratická funkce, parabola

$\boxed{\text{Př } 1}$ nakreslete $f(x) = (x-3)^2+1$



obecně $f(x) = ax^2+bx+c$, $a,b,c \in \mathbb{R}$, $a \neq 0$

Pozn. $a>0 \dots \cup$

$a<0 \dots \cap$

1) průsečík s osou y … $x=0$ … $P_y=[0,c]$ (existuje vždy)
2) průsečík s osou x … $ax^2+bx+c=0$ „řešení kvadratické rce“

2 způsoby A), B)

A) <u>Diskriminant</u> (vždy provedte zkoušku)

$D = b^2-4ac$ $\qquad x_{1,2} = \dfrac{-b \pm \sqrt{D}}{2a}$

$D>0$ … 2 kořeny (průsečíky) $\qquad P_{x_1}=[x_1,0]$

$D=0$ … 1 kořen $\qquad P_{x_2}=[x_2,0]$

$D<0$ … žádný kořen

$\cup$ nebo

$y=0$

$\cap$

B) Vietovy vztahy

! platí pro tvar $x^2+bx+c=0$ $(a=1)$

$x_1 \cdot x_2 = c$

$x_1 + x_2 = -b$

našel jsem kořeny x_1, x_2, mohu zapsat f v součinovém tvaru

$ax^2+bx+c = a(x-x_1)(x-x_2)$

3) souřadnice vrcholu

A) doplnění na čtverec

$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x\right)+c =$$
$$= a\left(x^2+\frac{b}{a}x + \boxed{\left(\frac{b}{2a}\right)^2}\right) \boxed{-\frac{b^2}{4a}} + c =$$
$$= a\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c =$$
$$= a\left(x-\left(-\frac{b}{2a}\right)\right)^2 - \frac{b^2}{4a} + c$$

$$V = \left[-\frac{b}{2a}; c - \frac{b^2}{4a}\right]$$

$$\| $$

$$\frac{-b^2-4ac}{4a} = \frac{-D}{4a}$$

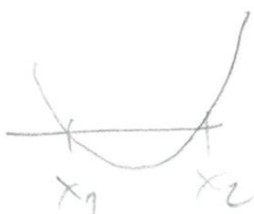
B) vzorcem: stačí si zapamatovat $V = \left[-\frac{b}{2a}; \dots\right]$

dosadit dosazením

C) derivace, najít extrém

4) Kdy je $f(x) \overset{?}{\substack{<0 \\ >0}}$

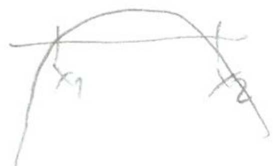
a) mám 2 kořeny: $a>0$



$f(x)<0$

$x \in (x_1, x_2)$

$a<0$



$f(x)>0$

$x \in (x_1, x_2)$

b) mám 1, žádný kořen $a>0 \Rightarrow f(x) \geq 0$, $a<0 \Rightarrow f(x) \leq 0$

$\boxed{Př}\ \boxed{2}$ $\quad -\frac{1}{2}x^2 + \frac{5}{2}x - 3$

1) $P_y = [0, -3]$

2) B) $-\frac{1}{2}x^2 + \frac{5}{2}x - 3 = 0$

$x^2 - 5x + 6 = 0$

$x_1 \cdot x_2 = 6$ $\quad\rightsquigarrow\quad$ $x_1 = 3$ $\quad P_{x_1} = [2, 0]$

$x_1 + x_2 = 5$ $\quad\quad\quad\quad\ $ $x_2 = 2$ $\quad P_{x_2} = [3, 0]$

A) $D = 25 - 4 \cdot 6 = 1$

$x_{1,2} = \frac{5 \pm 1}{2} = \left\langle \begin{matrix} 3 \\ 2 \end{matrix} \right.$

3) $\tilde{A}$) $-\frac{1}{2}(x^2 - 5x) - 3 = -\frac{1}{2}\left(x^2 - 5x + \frac{5^2}{2^2}\right) + \frac{5^2}{2^3} - 3 =$

$= -\frac{1}{2}\left(x - \frac{5}{2}\right)^2 + \frac{25}{8} - \frac{24}{8} =$

$= -\frac{1}{2}\left(x - \frac{5}{2}\right)^2 + \frac{1}{8}$

$V = \left[\frac{5}{2}; \frac{1}{8}\right]$

$\tilde{B}$) $V = \left[\frac{-\frac{5}{2}}{-1}; -\frac{1}{2} \cdot \frac{5^2}{2^2} + \frac{5}{2} \cdot \frac{5}{2} - 3\right]$

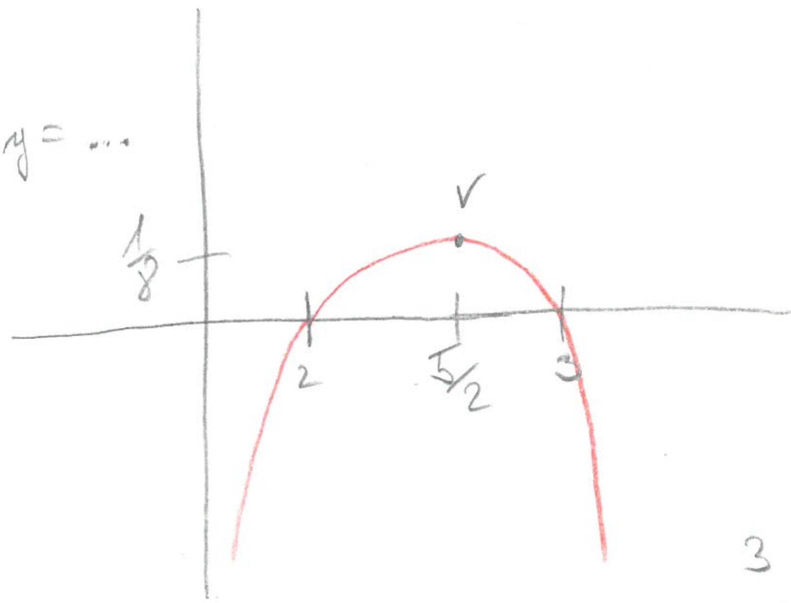
$\frac{-\frac{5}{2}}{-1} = \frac{5}{2}$

$-\frac{1}{2} \cdot \frac{5^2}{2^2} + \frac{5}{2} \cdot \frac{5}{2} - 3 = -\frac{25}{8} + \frac{25}{4} - \frac{24}{8} = \frac{50 - 25 - 24}{8} = \frac{1}{8}$

$\tilde{C}$) $\left(-\frac{1}{2}x^2 + \frac{5}{2}x - 3\right)' = 2 \cdot \left(-\frac{1}{2}\right)x + \frac{5}{2}$

$-x + \frac{5}{2} = 0$

$x = +\frac{5}{2}$, $y = \dots$



Př $\quad 3x^2-6x-105 = 3(x+5)(x-7)$

$P_y = [0, -105]$

$P_x = \{[-5, 0], [7, 0]\}$

$V = [1, 3-6-105] = [1, -108]$

Př $\quad \frac{1}{3}x^2+x+\frac{2}{3} = \frac{1}{3}(x+2)(x+1)$

$P_y = [0, \frac{2}{3}]$

$P_x = \{[-2, 0], [-1, 0]\}$

$V = \left[-\frac{3}{2}; \frac{3}{4}-\frac{3}{2}+\frac{2}{3}\right] = \left[-\frac{3}{2}; \frac{9+8-18}{12}\right] = \left[-\frac{3}{2}; -\frac{1}{12}\right]$

Př $\quad -4x^2+20x+24 = -4(x+1)(x-6)$

$P_y = [0, 24]$

$P_x = \{[-1; 0], [6, 0]\}$

$V = \left[\frac{5}{2}; -4\cdot\frac{25}{4}+20\cdot\frac{5}{2}+24\right] =$

$= \left[\frac{5}{2}; -25+50+24\right] = \left[\frac{5}{2}; 49\right]$

Př $\quad -x^2+6x-10$

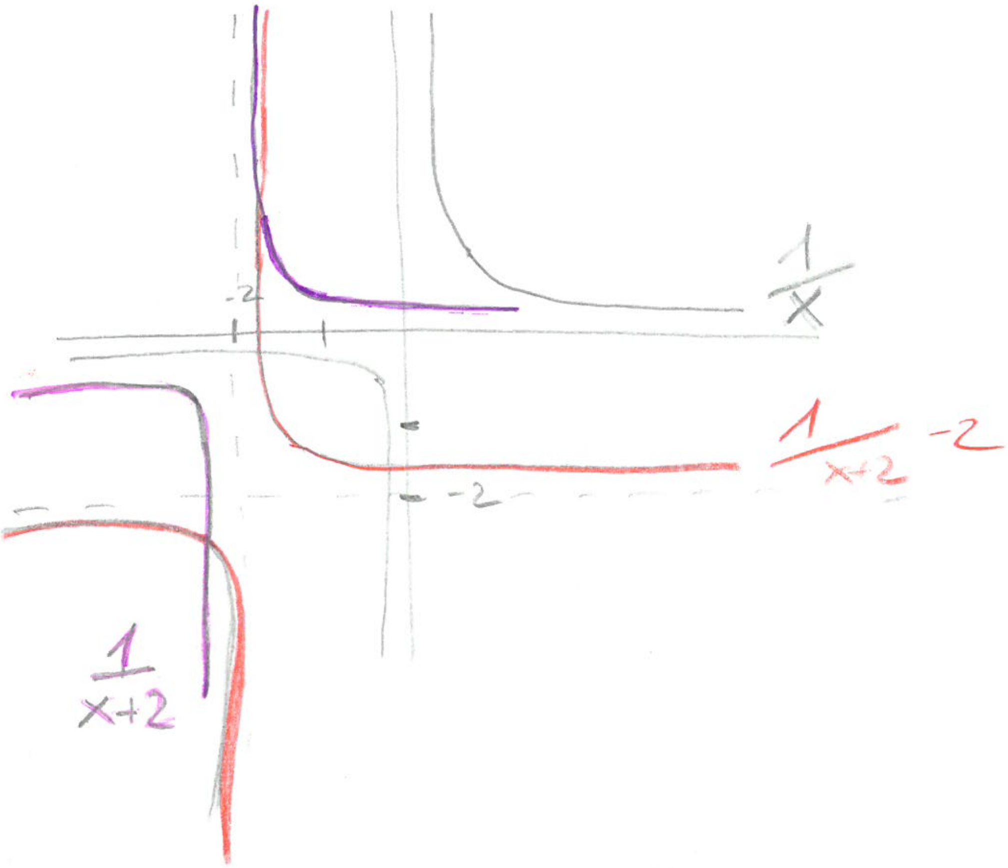
$P_y = [0; -10]$

nemá kořeny

$V = \boxed{[3; -1]}$

II Lineární lomené funkce

$\boxed{Př}$ nakreslete $f(x) = \dfrac{1}{x+2} - 2$



$S = [-2; -2]$

$\boxed{Př}$ $f(x) = \dfrac{5x+2}{8x+6}$ průsečíky? střed? asymptoty? nákres

$8x+6=0$

$x = -\frac{6}{8} = -\frac{3}{4}$

$5x+2 : (8x+6) = \frac{5}{8} + \dfrac{-\frac{7}{4}}{8x+6}$

$5x + \frac{30}{8}$

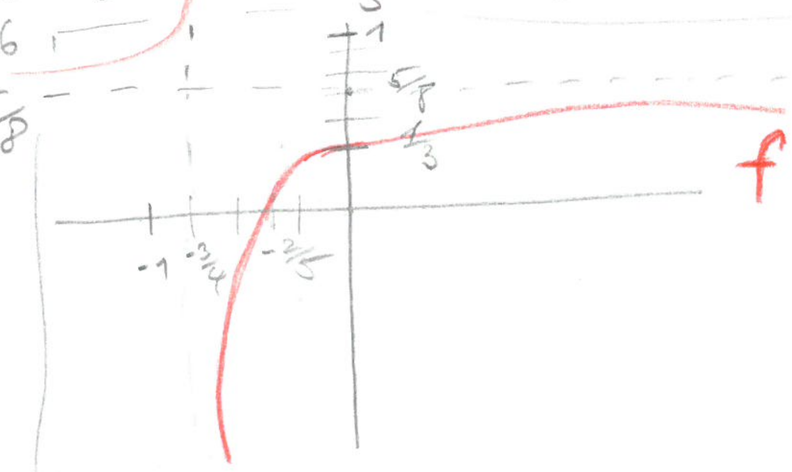
$\frac{8}{4} - \frac{15}{4}$

$S = [-\frac{3}{4}; \frac{5}{8}]$

Průsečík s osou y : $f(0) = \frac{2}{6} \dots [0, \frac{1}{3}]$

—〃— x : $\dfrac{5x+2}{8x+6} = 0$ $x = -\frac{2}{5} \dots [-\frac{2}{5}; 0]$

Asymptoty: $x = -\frac{3}{4}$, $y = \frac{5}{8}$



$\boxed{P_{12}}$ $\dfrac{-2x+13}{2x-6}$

$P_y = [0, -\frac{13}{6}]$

$P_x = [\frac{13}{2}; 0]$

$S = [3; -1]$

$-2x+13 : 2x-6 = -1 + \dfrac{7}{(2x-6)}$

$-2x + 6$

$13-6$

$(2(x-3))$

$\boxed{P_{12}^{*}}$ $\dfrac{\sqrt{2}x - \sqrt{6} + 1}{x - \sqrt{3}}$

$P_y = [0, \frac{-\sqrt{6}+1}{-\sqrt{3}}] = [0, \frac{\sqrt{18} - \sqrt{3}}{3}] = [0, \frac{3\sqrt{2}-\sqrt{3}}{3}]$

$P_x = [\frac{\sqrt{6}-1}{\sqrt{2}}; 0] = [\frac{2\sqrt{3}-\sqrt{2}}{2}; 0]$

$S = [\sqrt{3}, \sqrt{2}]$

$\sqrt{2}x - \sqrt{6} + 1 : x - \sqrt{3} = \sqrt{2} + \dfrac{1}{x-\sqrt{3}}$

$\sqrt{2}x - \sqrt{2}\sqrt{3} + 1$