Posloupnost „postupně očíslovaná reálná čísla"

$\boxed{CV4}$

$a_1 = 2,\ a_2 = 3,\ a_3 = 4,\ a_4 = 5, \dots$

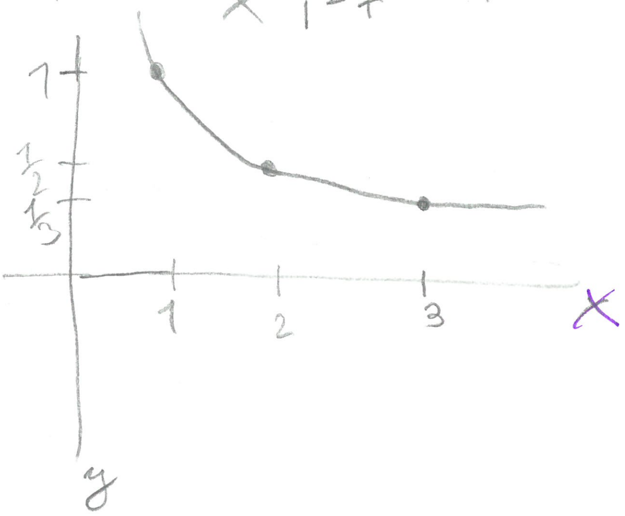
značení $\{a_n\}_{n=1}^{\infty}$

zadání vzorcem – např. $a_n = n^2,\ n \in \mathbb{N}$

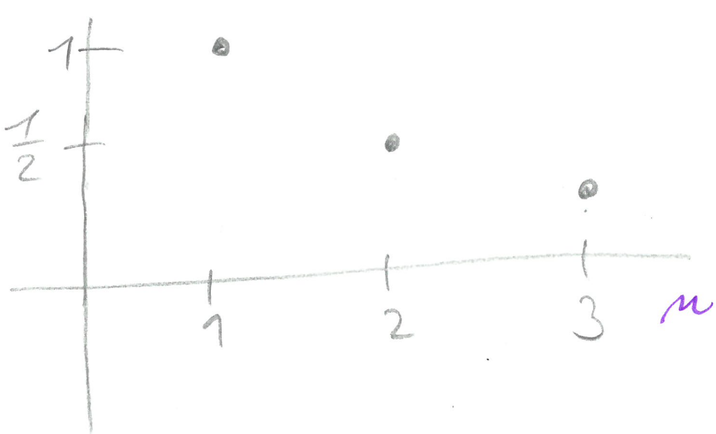
$(a_1 = 1, a_2 = 4, a_3 = 9, \dots)$

funkce vs. posloupnost

$f(x) = \dfrac{1}{x},\ D_f = (0; +\infty)$



$a_n = \dfrac{1}{n},\ n \in \mathbb{N}$



limita posloupnosti

Posloupnost $\{a_n\}_{n=1}^{\infty}$ má (v ∞) limitu a, pokud se členy posloupnosti blíží libovolně blízko k a, když zvětšujeme n.

značení $\lim\limits_{n \to \infty} a_n = a$

$a_n \to a$

Známé limity:

- $\lim n = +\infty$
- $\lim \frac{1}{n} = 0$

obecně ■ $\lim n^{\alpha} = \begin{cases} 0, & \alpha < 0 \\ 1, & \alpha = 0 \\ \infty, & \alpha > 0 \end{cases}$

$\boxed{\text{Př}}$ $\lim n^3 = \infty$ $\boxed{\text{Př}}$ $\lim n^{-2} = 0$ $\boxed{\text{Př}}$ $\lim \frac{1}{n^3} = 0$

- $\lim 2^n = \infty$
- $\lim 3^{-n} = 0$

obecně ■ $\lim a^n = \begin{cases} \text{neex.}; & a \leq -1 \\ 0; & a \in (-1;1) \\ 1; & a = 1 \\ \infty; & a > 1 \end{cases}$

$\boxed{\text{Př}}$ $\lim (-1)^n$ neex, $\boxed{\text{Př}}$ $\lim \frac{1}{3}^n = 0$ $\boxed{\text{Př}}$ $\lim e^n = \infty$

Věty o aritmetice limit (VOAL)

$$\left.\begin{aligned} \lim a_n \pm b_n &= \lim a_n \pm \lim b_n \\ \lim a_n \cdot b_n &= (\lim a_n) \cdot (\lim b_n) \\ \lim \frac{a_n}{b_n} &= \frac{\lim a_n}{\lim b_n} \end{aligned}\right\}$$

POKUD MÁ PRAVÁ STRANA SMYSL

Nedobinorové výrazy:

■ $\infty - \infty$

■ $\frac{0}{\pm\infty}$

■ $\frac{\pm\infty}{\pm\infty}$

■ $+\infty \cdot 0$

■ $\frac{\pm\infty}{0}, \frac{a}{0}, \frac{0}{0}$

$\boxed{\text{Př}}$ $\lim n^2 + 6n + 2 \overset{\text{VOAL}}{\underset{?}{=}} \lim n^2 + \lim 6n + \lim 2 \overset{\text{VOAL}}{\underset{?}{=}} \infty + \infty + 2 \underset{?}{=} +\infty$

$\boxed{\text{Př}}$ $\lim n^2 - 6n + 2 \overset{\text{VOAL}}{\underset{?}{=}} +\infty - \infty = ?$

$$\lim n^2 - 6n + 2 = \lim n^2 \left(1 - \frac{6n}{n^2} + \frac{2}{n^2}\right) =$$

VYTKNOUT N NA NEJVYŠŠÍ MOCNINU

$\boxed{\text{FINTA Č.1}}$

$$\overset{VOAL}{\underset{?}{=}} \lim n^2 \cdot \lim \left(1 - \frac{6\cancel{n}}{n^{\cancel{2}\,1}} + \frac{2}{n^2}\right) \overset{VOAL}{\underset{?}{=}}$$

$$= \lim n^2 \cdot \left[\lim 1 - \lim \frac{6}{n} + \lim \frac{2}{n^2}\right] =$$

$$= +\infty \cdot [1 - 6 \cdot 0 + 2 \cdot 0] = +\infty \cdot 1 =$$

$$= +\infty$$

Př $\lim \frac{n+5}{5n-25} = ?$

$$\lim \frac{n+5}{5n-25} = \lim \frac{\cancel{n}\left(1+\frac{5}{n}\right)}{\cancel{n}\left(5-\frac{25}{n}\right)} = \lim \frac{\left(1+\frac{5}{n}\right)}{\left(5-\frac{25}{n}\right)} =$$

VYTKNOUT
N NA
NEJVYŠŠÍ
MOCNINU

ZKRÁTIT

$$\overset{\text{VOAL}}{\underset{?}{=}} \frac{\lim\left(1+\frac{5}{n}\right)}{\lim\left(5-\frac{25}{n}\right)} \overset{\text{VOAL}}{\underset{?}{=}} \frac{\lim 1 + \lim \frac{5}{n}}{\lim 5 - \lim \frac{25}{n}} \underset{?}{=}$$

$$\underset{?}{=} \frac{\lim 1 + 5\cdot \lim \frac{1}{n}}{\lim 5 - 25\cdot \lim \frac{1}{n}} \overset{?}{=} \frac{1+5\cdot 0}{5-25\cdot 0} = \boxed{\frac{1}{5}}$$

lim KONSTANTA = KONSTANTA
$\lim \frac{1}{n} = 0$

Př $\lim \frac{n^3+4n^2-1}{(n-1)^3+(3n-2)^2} =$

UPRAVIT

$$= \lim \frac{n^3+4n^2-1}{n^3-3n^2+3n-1+9n^2-12n+4} =$$

$$= \lim \frac{n^3+4n^2-1}{n^3+6n^2-9n+3} \overset{\text{VYTKNUTÍ}}{=} \lim \frac{\cancel{n^3}\left(1+\frac{4n^2}{n^3}-\frac{1}{n^3}\right)}{\cancel{n^3}\left(1+\frac{6n^2}{n^3}-\frac{9n}{n^3}+\frac{3}{n^3}\right)} =$$

ZKRÁTIT

ZNÁMÉ LIMITY
$\lim \frac{1}{n} = 0$
$\lim \frac{1}{n^2} = 0$
$\lim \frac{1}{n^3} = 0$

$$= \lim \frac{\left(1+\frac{4}{n}-\frac{1}{n^3}\right)}{\left(1+\frac{6}{n}-\frac{9}{n^2}+3\right)} \overset{\text{VOAL}}{\underset{?}{=}} \frac{\lim(\quad)}{\lim(\quad)} \overset{\text{VOAL}}{\underset{?}{=}}$$

$$= \frac{\lim 1 + \lim\frac{4}{n} - \lim\frac{1}{n^3}}{\lim 1 + \lim\frac{6}{n} - \lim\frac{9}{n^2} + \lim\frac{3}{n^3}} \underset{?}{=} \frac{1+4\cdot 0-0}{1+6\cdot 0-9\cdot 0+3\cdot 0} = \boxed{1}$$

$\boxed{\text{Př}}$

$\boxed{\text{FINTA Č.3}}$

$$\lim \sqrt{n}\,(\overset{a}{\sqrt{n+2}} - \overset{b}{\sqrt{n}}) \cdot \underbrace{\boxed{\frac{(\overset{a}{\sqrt{n+2}} + \overset{b}{\sqrt{n}})}{(\sqrt{n+2}+\sqrt{n})}}}_{=1} =$$

VHODNÉ PŘENÁSOBENÍ

POUŽITÍ VZORCE $(a-b)(a+b)=a^2-b^2$

$$= \lim \frac{\sqrt{n}\,(n+2-n)}{\sqrt{n+2}+\sqrt{n}} \underset{\boxed{\text{VYTKNUTÍ}}}{=} \lim \frac{2\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}\left(\sqrt{1+\frac{2}{n}}+1\right)} =$$

$$\overset{?}{\underset{\text{VOAL}}{=}} \frac{\lim 2}{\lim \sqrt{1+\frac{2}{n}}+1} = \frac{\lim 2}{\sqrt{\lim 1 + \lim \frac{2}{n}} + \lim 1} =$$

$$= \frac{2}{\sqrt{1+0}+1} = \frac{2}{2} = \boxed{1}$$

$\boxed{\text{Př}}$

$\boxed{\text{FINTA Č.2}}$

$$\lim \frac{3^{n-1}+7^n}{7^{n-1}+5^n} \underset{\boxed{\text{VYTKNOUT NEJVYŠŠÍ ZÁKLAD}}}{=} \lim \frac{\cancel{7^n}\left(\frac{1}{3}\cdot\frac{3^n}{7^n}+1\right)}{\cancel{7^n}\left(\frac{1}{7}+\frac{5^n}{7^n}\right)} =$$

$\boxed{\text{ZKRÁTIT}}$

$$\overset{\text{VOAL}}{\underset{?}{=}} \frac{\frac{1}{3}\lim\left(\frac{3}{7}\right)^n + \lim 1}{\lim \frac{1}{7} + \lim \left(\frac{5}{7}\right)^n} = \frac{\frac{1}{3}\cdot 0 + 1}{\frac{1}{7}+0} = \frac{1}{\frac{1}{7}} = 7$$

ZNÁMÉ LIMITY

$$\lim \left(\frac{3}{7}\right)^n = 0$$

$$\lim \left(\frac{5}{7}\right)^n = 0$$

$\boxed{P\bar{r}}$ $$\lim \frac{(n+2)^3 - n(n^2+1)}{(4-3n)^2} = \frac{2}{3}$$

$\boxed{P\bar{r}}$ $$\lim \frac{6n^4 + n}{-9n^3 - 2n^2} = -\infty$$