$\boxed{P_2}$ $f(x) = \dfrac{\sqrt{x-2}}{-x^2+2x+3}$

1) D_f: a) $x-2 \geq 0 \rightarrow x \geq 2$, $x \in \langle 2; +\infty)$

b) $-x^2+2x+3 \neq 0$

$x^2-2x-3=0$

$-1 \quad 3 \quad 2$ $\rightarrow x_1 = -1$, $x_2 = 3$

celkom $D_f = \langle 2;3) \cup (3;+\infty)$

2) P_y – neexistuje $x=0 \in D_f$

P_x: $\dfrac{\sqrt{x-2}}{-x^2+2x+3} = 0 \Leftrightarrow \sqrt{x-2} = 0 \Leftrightarrow x-2=0$

$x = 2$

$P_x = [2;0]$

3) f(x) KLADNÁ/ZÁPORNÁ

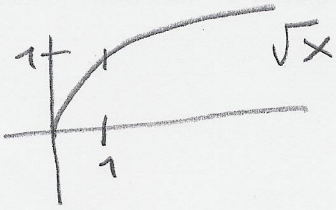
$\dfrac{\sqrt{x-2}}{-x^2+2x+3} = \dfrac{\sqrt{x-2}}{-(x+1)(x-3)}$

$x \in$	$\langle 2;3)$	$(3;+\infty)$
$\sqrt{x-2}$	+	+
$(x+1)$	+	+
$(x-3)$	−	+
−	−	−
$f(x)$	+	−

$f(x) \geq 0$; $x \in \langle 2;3)$

$f(x) \leq 0$; $x \in (3;+\infty)$

$\boxed{\text{Př}}$ $\lim \sqrt{n} = +\infty$



$\sqrt{1} = 1$

$\sqrt{0} = 0$

$\boxed{\text{Př}}$

$$\lim \underbrace{\sqrt{\underbrace{n^2+4}_{\downarrow\ \infty}}}_{\to +\infty} - \underbrace{\sqrt{\underbrace{n^2-4}_{\to +\infty}}}_{\to +\infty} = +\infty - \infty$$

NEDEF VÝRAZ

FINTA Č.3 VHODNÉ PŘENÁSOBENÍ

$$\lim \underbrace{\sqrt{n^2+4}}_{a} - \underbrace{\sqrt{n^2-4}}_{b} \cdot \frac{\overbrace{\sqrt{n^2+4}}^{a} + \overbrace{\sqrt{n^2-4}}^{b}}{\sqrt{n^2+4} + \sqrt{n^2-4}} \underset{\uparrow}{=}$$

$(a-b)(a+b) = a^2 - b^2$

$$= \lim \frac{\left(\sqrt{n^2+4}\right)^2 - \left(\sqrt{n^2-4}\right)^2}{\sqrt{n^2+4} + \sqrt{n^2-4}} = \lim \frac{\cancel{n^2}+4-\cancel{n^2}+4}{\sqrt{n^2+4} + \sqrt{n^2-4}} =$$

$$= \lim \frac{8}{\underbrace{\sqrt{n^2+4}}_{\to +\infty} + \underbrace{\sqrt{n^2-4}}_{\to +\infty}} \underset{\uparrow}{=} \frac{8}{+\infty+\infty} = \frac{8}{+\infty} \underset{\uparrow}{=} 0$$

LIMITNÍ PŘECHOD

$\dfrac{\text{KONSTANTA}}{+\infty}$

$$\lim \sqrt{n}\left(\sqrt{n+2}-\sqrt{n}\right) = +\infty(+\infty-\infty)$$

NEDEF. VÝRAZ

$$\lim \sqrt{n} = +\infty$$

FINTA Č.3 - VHODNÉ PŘENÁSOBENÍ $\sqrt{n} = n^{1/2}$

$$\lim \sqrt{n}\left(\underbrace{\sqrt{n+2}}_{a}-\underbrace{\sqrt{n}}_{b}\right)\cdot\frac{\sqrt{n+2}+\sqrt{n}}{\underbrace{\sqrt{n+2}}_{a}+\underbrace{\sqrt{n}}_{b}} =$$

$(a-b)(a+b) = a^2-b^2$

$$= \lim \sqrt{n}\cdot\frac{\left(\sqrt{n+2}\right)^2-\left(\sqrt{n}\right)^2}{\sqrt{n+2}+\sqrt{n}} = \lim \frac{\sqrt{n}\left[n+2-n\right]}{\sqrt{n+2}+\sqrt{n}} =$$

$$= \lim \frac{2\sqrt{n}}{\sqrt{n+2}+\sqrt{n}} = \frac{2\cdot(+\infty)}{+\infty+\infty} = \frac{+\infty}{+\infty}$$

NEDEF. VÝRAZ

ZKUSÍM POUŽÍT LIMITNÍ PŘECHOD

FINTA Č.1

$$\boxed{\begin{array}{l}\text{VYTÝKÁM ZPOD ODMOCNINY}\\ \sqrt{n+2} = \sqrt{n\left(1+\frac{2}{n}\right)} = \sqrt{n}\cdot\sqrt{1+\frac{2}{n}} = \\ \qquad = n^{1/2}\cdot\sqrt{1+\frac{2}{n}}\end{array}}$$

$$\lim \frac{2n^{1/2}}{n^{1/2}\cdot\sqrt{1+\frac{2}{n}}+n^{1/2}} = \lim \frac{2\cancel{n^{1/2}}}{\cancel{n^{1/2}}\left[\underbrace{\sqrt{1+\frac{2}{n}}}_{\sqrt{1}=1}+1\right]} = \frac{2}{1+1} = 2$$

$\frac{2}{n} \to 0$

LIMITNÍ PŘECHOD

$$\lim \frac{3\cdot 3^{2n} - \frac{1}{(23)^n} + 3\cdot\left(\frac{9}{2}\right)^{2n}}{8\cdot 9^n - \left(\frac{1}{3}\right)^{5n} + 5(4{,}5)^{2n}} =$$

$$= \lim \frac{\underbrace{3\cdot 9^n}_{\to +\infty} - \underbrace{\frac{1}{(23)^n}}_{\to 0} + \underbrace{3\cdot\left(\frac{81}{4}\right)^n}_{\to +\infty}}{\underbrace{8\cdot 9^n}_{\to +\infty} - \underbrace{\left(\frac{1}{3^5}\right)^n}_{\to 0} + \underbrace{5\cdot\left(\frac{81}{4}\right)^n}_{\to +\infty}} = \frac{+\infty}{+\infty}$$ NEDEFINOVANÝ VÝRAZ

DOM. ČLEN $\left(\frac{81}{4} \approx 20 > 9\right)$

FINTA Č. 2 – VYTKNUTÍ DOM. ČLENU

$$= \lim \frac{\cancel{\left(\frac{81}{4}\right)^n}\left[3\cdot\frac{9^n}{\left(\frac{81}{4}\right)^n} - \frac{\frac{1}{(23)^n}}{\left(\frac{81}{4}\right)^n} + 3\right]}{\cancel{\left(\frac{81}{4}\right)^n}\left[8\cdot\frac{9^n}{\left(\frac{81}{4}\right)^n} - \frac{\left(\frac{1}{3^5}\right)^n}{\left(\frac{81}{4}\right)^n} + 5\right]} =$$

$$= \lim \frac{\left[3\cdot\underbrace{\left(\frac{4}{81_9}\cdot 9\right)^n}_{\to 0} - \underbrace{\left(\frac{4}{81}\cdot\frac{1}{23}\right)^n}_{\to 0} + 3\right]}{\left[8\cdot\underbrace{\left(\frac{4}{81_9}\cdot 9_1\right)^n}_{\to 0} - \underbrace{\left(\frac{1}{3^5}\cdot\frac{81}{4}\right)^n}_{\to 0} + 5\right]} = \frac{3\cdot 0 - 0 + 3}{8\cdot 0 - 0 + 5} = \frac{3}{5}$$

$\boxed{\text{Př}}$

$$\lim \frac{\left(\frac{3}{4}\right)^{2n-1} - \left(\frac{2}{6}\right)^{n+1}}{\left(\frac{18}{32}\right)^{n} + \left(\frac{1}{23}\right)^{n}} =$$

$$\left(\frac{3}{4}\right)^{2n-1} = \left(\frac{3}{4}\right)^{2n} \cdot \left(\frac{3}{4}\right)^{-1} = \left(\frac{9}{16}\right)^{n} \cdot \frac{4}{3}$$

$$\left(\frac{2}{6}\right)^{n+1} = \left(\frac{1}{3}\right)^{n+1} = \left(\frac{1}{3}\right)^{n} \cdot \frac{1}{3}$$

$$= \lim \frac{\underbrace{\left(\frac{9}{16}\right)^{n}}_{\to 0} \cdot \frac{4}{3} - \underbrace{\left(\frac{1}{3}\right)^{n}}_{\to 0} \cdot \frac{1}{3}}{\underbrace{\left(\frac{9}{16}\right)^{n}}_{\to 0} + \underbrace{\left(\frac{1}{23}\right)^{n}}_{\to 0}} =$$

$$= \frac{0 - 0}{0 + 0} = \frac{0}{0}$$ NEDEFINOVANÝ VÝRAZ

FINTA Č.2 $\left(\frac{9}{16} > \frac{1}{3}\right)$

$$\lim \frac{\left(\frac{9}{16}\right)^{n} \cdot \frac{4}{3} - \left(\frac{1}{3}\right)^{n} \cdot \frac{1}{3}}{\left(\frac{9}{16}\right)^{n} + \left(\frac{1}{23}\right)^{n}} = \lim \frac{\cancel{\left(\frac{9}{16}\right)^{n}} \left[\frac{4}{3} - \frac{1}{3} \cdot \frac{\left(\frac{1}{3}\right)^{n}}{\left(\frac{9}{16}\right)^{n}}\right]}{\cancel{\left(\frac{9}{16}\right)^{n}} \left[1 + \frac{\left(\frac{1}{23}\right)^{n}}{\left(\frac{9}{16}\right)^{n}}\right]} =$$

$$= \lim \frac{\frac{4}{3} - \frac{1}{3} \cdot \left(\frac{1}{3} \cdot \frac{16}{9}\right)^{n}}{1 + \left(\frac{1}{23} \cdot \frac{16}{9}\right)^{n}} = \lim \frac{\frac{4}{3} - \frac{1}{3} \underbrace{\left(\frac{16}{27}\right)^{n}}_{\to 0}}{1 + \underbrace{\left(\frac{16}{23 \cdot 9}\right)^{n}}_{\to 0}} =$$

$\frac{16}{27} < 1$

$\frac{16}{23 \cdot 9} < 1$

$$= \frac{4/3}{1} = \frac{4}{3}$$

$\uparrow$ LIMITNÍ PŘECHOD

$\boxed{P_{\overline{R}}}$ FINTA $\bar{c}$.3

$$\lim \frac{\sqrt{4n^3+2n^2-3}-\sqrt{4n^3-n^2+2}}{\sqrt{4n}} \cdot \frac{\sqrt{4n^3+2n^2-3}+\sqrt{4n^3-n^2+2}}{-\!\!\!-\,''\,-\!\!\!-}$$

$$= \lim \frac{3n^2-5}{\sqrt{4n}\left(\sqrt{4n^3+2n^2-3}+\sqrt{4n^3-n^2+2}\right)} =$$

$$= \lim \frac{3n^2\left(1-\frac{5}{3n^2}\right)}{\sqrt{16n^4}\left(\sqrt{1+\frac{2n^2}{4n^3}-\frac{3}{4n^3}}+\sqrt{1-\frac{n^2}{4n^3}+\frac{2}{4n^3}}\right)} =$$

$$= \lim \frac{3\cancel{n^2}\left(1-\underbrace{\frac{5}{3n^2}}_{\to 0}\right)}{4\cancel{n^2}\left(\sqrt{1+\underbrace{\frac{2n^2}{4n^3}}_{0}-\underbrace{\frac{3}{4n^3}}_{0}}+\sqrt{1-\underbrace{\frac{n^2}{4n^3}}_{0}+\underbrace{\frac{2}{4n^3}}_{0}}\right)} =$$

$$\overset{\text{VOAL}}{=} \frac{3\cdot(1-0)}{4\left(\sqrt{1+0-0}+\sqrt{1-0+0}\right)} = \frac{3}{8}$$

Př

$$\lim_{n\to\infty} \sqrt{n^2+3n} - n = \frac{3}{2}$$

FINTA Č.3

Př

$$\lim 2n - \sqrt{4n^2+7n} = -\frac{7}{4}$$

Př

$$\lim_{n\to\infty} \frac{2\cdot 3^n + 3\cdot 2^n}{2^{n-1} - 3^{n+1}} = -\frac{2}{3}$$

Př

$$\lim_{n\to\infty} \frac{5\cdot 4^{n-1} + 3\cdot 2^{n+1}}{4^n - 2^{n+6}} = \frac{5}{4}$$

FINTA Č.2

Př

$$\lim \frac{\left(\frac{1}{7}\right)^n + \left(\frac{1}{2}\right)^{n+1}}{\left(\frac{1}{4}\right)^n - \left(\frac{1}{2}\right)^{n+2}} = -2$$