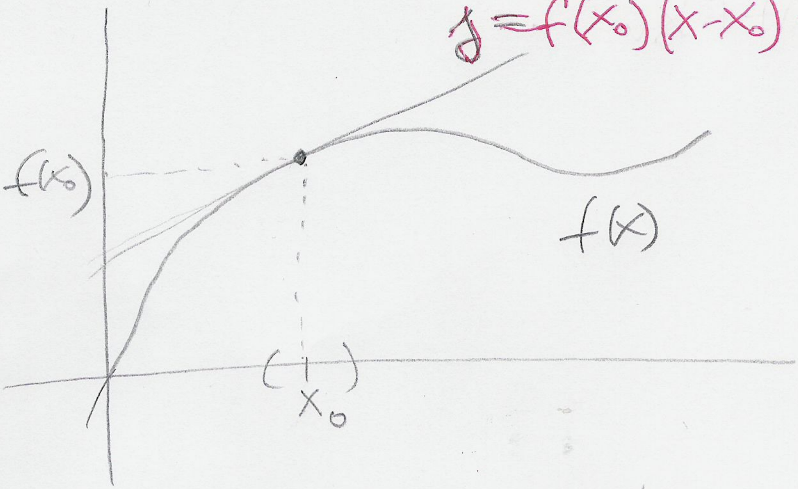
Derivace funkce

CV7

$y = f'(x_0)(x-x_0) + f(x_0)$



$f(x)$, $x \in D_f$ ... funkce f

„v malém okolí bodu x_0 lze f nahradit lineární funkcí

$y = f'(x_0)(x-x_0) + f(x_0)$ "

Derivace základních funkcí

1) $f(x) = c$, $f'(x) = 0$, $c \in \mathbb{R}$ - konst.

2) $f(x) = x^{\alpha}$, $f'(x) = \alpha x^{(\alpha-1)}$, $\alpha \in \mathbb{R}$

3) $f(x) = e^{x}$, $f'(x) = e^{x}$

4) $f(x) = a^{x}$, $f'(x) = a^{x} \cdot \ln a$, $a > 0$

5) $f(x) = \ln x$, $f'(x) = \dfrac{1}{x}$

6) $f(x) = \log_a x$, $f'(x) = \dfrac{1}{x \ln a}$, $a > 0$

Př

$(94)' = 0$
$(x^{23})' = 23 \cdot x^{22}$
$(e^{x})' = e^{x}$
$(15^{x})' = 15^{x} \ln 15$
$(\ln x)' = \dfrac{1}{x}$
$(\log_6 x)' = \dfrac{1}{x \ln 6}$

Pravidla pro počítání s derivacemi

a) $(f \pm g)' = f' + g'$

b) $(f \cdot g)' = f'g + fg'$

speciálně $(c \cdot f)' = c \cdot f'$ „vytknutí konstanty"

c) $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{(g)^2}$

d) $(f(g(x)))' = f'(g(x)) \cdot g'(x)$

„derivace složené funkce"

$\boxed{Př}$ $(5x^4)' = 5(x^4)' = 5 \cdot 4x^3 = 20x^3$

$\boxed{Př}$ $(6x^4 + 3x^3 - 2x + 10)' \underset{a)}{=}$

$= (6x^4)' + (3x^3)' - (2x)' + (10)' \underset{b)}{=}$

$= 6(x^4)' + 3(x^3)' - 2(x)' + (10)' =$

$= 6 \cdot 4x^3 + 3 \cdot 3 \cdot x^2 - 2 \cdot 1x^0 + 0 =$

$= 24x^3 + 9x^2 - 2$

$\boxed{Př}$ $(\underbrace{(3x^2+5x^3) \cdot \ln x}_{f})' \underset{b)}{=} (3x^2+5x^3)' \cdot \ln x + (3x^2+5x^3)(\ln x)' =$

$= (6x + 15x^2) \ln x + (3x^2+5x^3) \cdot \frac{1}{x} =$

$= (6x + 15x^2) \ln x + 3x + 5x^2$

$D_f = (0; +\infty)$
$D_{f'} = (0; +\infty)$

$\boxed{Př}$ $\left(\underbrace{\frac{14x^2-3x+1}{x+1}}_{f}\right)' \underset{c)}{=} \frac{(14x^2-3x+1)'(x+1) - (14x^2-3x+1) \cdot (x+1)'}{(x+1)^2} =$

$= \frac{(28x-3)(x+1) - (14x^2-3x+1) \cdot (1)}{(x+1)^2} =$

$= \frac{28x^2 - 3x + 28x - 3 - 14x^2 + 3x - 1}{(x+1)^2} =$

$= \frac{14x^2 + 28x - 4}{(x+1)^2}$

$D_f = \mathbb{R} \setminus \{-1\}$
$D_{f'} = \mathbb{R} \setminus \{-1\}$

$\boxed{\text{Př}}$ $\left((\ln x)^2\right)' = 2(\ln x)^1 \cdot \frac{1}{x} = \frac{2\ln x}{x}$

vnější fce

vnitřní fce

$$\boxed{\begin{aligned} D_f &= (0;+\infty) \\ D_{f'} &= (0;+\infty) \end{aligned}}$$

$\boxed{\text{Př}}$ $\left(e^{(x^5+3x+1)}\right)' = e^{(x^5+3x+1)} \cdot (5x^4+3)$

$D_f = \mathbb{R}$

$D_{f'} = \mathbb{R}$

$\boxed{\text{Př}}$ $\left(\sqrt{x}\right)' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ $\quad D_f = [0;+\infty)$, $D_{f'} = (0;+\infty)$

$\boxed{\text{Př}}$ $\left(\frac{1}{x^3}\right)' = \frac{(1)'x^3 - 1\cdot(x^3)'}{(x^3)^2} = \frac{0-3x^2}{x^6} = \frac{-3}{x^4}$

$D_f = \mathbb{R}\setminus\{0\}$

$D_{f'} = \mathbb{R}\setminus\{0\}$

$\boxed{\text{Př}}$ $\left(\ln\left(\sqrt[3]{x}\right)\right)' = \frac{1}{\sqrt[3]{x}} \cdot \left(x^{\frac{1}{3}}\right)' =$

$= x^{-\frac{1}{3}} \cdot \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3}\cdot x^{-1} = \frac{1}{3x}$

$D_f = (0;+\infty)$

$D_{f'} = (0;+\infty)$

$\boxed{\text{Př}}$ $\left(e^{\left(\frac{5-3x}{x+2}\right)}\right)'$, $D_f = \mathbb{R}\setminus\{-2\}$, $D_{f'} = \mathbb{R}\setminus\{-2\}$

$\|$

$e^{\left(\frac{5-3x}{x+2}\right)} \cdot \frac{-3(x+2)-(5-3x)(1)}{(x+2)^2} =$

$= e^{\left(\frac{5-3x}{x+2}\right)} \cdot \frac{-11}{(x+2)^2}$

$\boxed{P_{\bar{2}}}$

$$\left(\frac{e^x}{\sqrt{8x^2-1}}\right)' \qquad \begin{aligned} &8x^2-1 \geq 0 \\ &8x^2 \geq 1 \\ &x \in \left(-\infty; -\sqrt{\tfrac{1}{8}}\right] \cup \left[\sqrt{\tfrac{1}{8}}; +\infty\right) \end{aligned}$$

$$= \frac{e^x\sqrt{8x^2-1} - \dfrac{e^x}{2\sqrt{8x^2-1}} \cdot 16x}{(8x^2-1)} =$$

$$= \frac{e^x}{\sqrt{8x^2-1}} - \frac{e^x \cdot 16x}{2\,(8x^2-1)^{\frac{3}{2}}}$$

$\boxed{P_{\bar{2}}}$

$$\left(\frac{x+2}{\sqrt{x^2+1}}\right)' = \frac{\sqrt{x^2+1} - \dfrac{(x+2)\cdot\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}}{x^2+1} =$$

$$= \frac{x^2+1-x^2-2x}{(x^2+1)^{\frac{3}{2}}} = \frac{1-2x}{(x^2+1)^{\frac{3}{2}}}$$

$\boxed{P_2}$

① $$\left(\frac{x^3+2x^2+1}{-x+2}\right)' = \frac{(3x^2+4x)(-x+2)-(x^3+2x^2+1)(-1)}{(-x+2)^2} =$$

$$= \frac{-2x^3+4x^2+8x+1}{x^2-4x+4}$$, $D_f = \mathbb{R}\setminus\{2\}$

$D_{f'} = \mathbb{R}\setminus\{2\}$

② $$\left(\sqrt{3x^2+4x+5}\right)' = \frac{1}{2\sqrt{3x^2+4x+5}} \cdot (6x+4)$$

$3x^2+4x+5 > 0 \quad \forall x \in \mathbb{R}$

$\Rightarrow D_f = \mathbb{R}$

$D_{f'} = \mathbb{R}$

③ $$\left((10x^3+e^x)\cdot \ln x\right)' = (30x^2+e^x)\ln x + (10x^3+e^x)\frac{1}{x}$$

$D_f = (0,+\infty)$, $D_{f'} = (0,+\infty)$

④ $$\left(\ln(x^2+x)\right)' = \frac{1}{x^2+x}\cdot 2x+1 = \frac{2x+1}{x^2+x}$$

$D_f = (0,+\infty)$

$D_{f'} = (0,+\infty)$

⑤ $$\left((10x^5+x^2)\cdot e^x\right)' = (50x^4+2x)e^x + (10x^5+x^2)e^x =$$

$$= (10x^5+50x^4+x^2+2x)\,e^x$$