L'HOSPITALOVO PRAVIDLO

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \text{„}\frac{0}{0}\text{"} \text{ nebo } \text{„}\frac{\text{cokoliv}}{\pm\infty}\text{"}$$

pak $\displaystyle \lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$ MÁ-LI P.S. SMYSL

$\boxed{\text{Př}}$ $$\lim_{x \to 1} \frac{x^2-1}{x-1} \underset{\substack{L'H \\ „\frac{0}{0}"}}{=} \lim_{x \to 1} \frac{(x^2-1)'}{(x-1)'} = \lim_{x \to 1} \frac{2x}{1} = 2$$

$\boxed{\text{Př}}$ $$\lim_{x \to +\infty} \frac{x^2-3}{1-3x^2} \underset{\substack{L'H \\ „\frac{\infty}{\infty}"}}{=} \lim_{x \to \infty} \frac{2\cancel{x}}{-6\cancel{x}} = -\frac{1}{3}$$

$\boxed{\text{Př}}$ $$\lim_{x \to 1} \frac{\ln(x)}{1-x} \underset{\substack{L'H \\ „\frac{0}{0}"}}{=} \lim_{x \to 1} \frac{\frac{1}{x}}{-1} = \frac{\frac{1}{1}}{-1} = -1$$

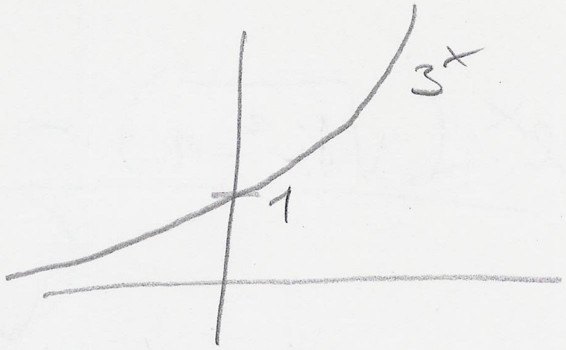
DOSADIT

$\boxed{\text{Př}}$ $$\lim_{x \to 1} \frac{e^x - e}{x^2-1} \underset{\substack{L'H \\ „\frac{0}{0}"}}{=} \lim_{x \to 1} \frac{e^x}{2x} = \frac{e^1}{2 \cdot 1} = \frac{e}{2}$$

DOSADIT

$\boxed{PŘ}$ $f(x) = (x+5)\,3^{2x-1}$

$D_f = (-\infty; +\infty)$



$\lim\limits_{x\to+\infty} 3^x = +\infty$

$\lim\limits_{x\to-\infty} 3^x = 0$

A) $\lim\limits_{x\to+\infty} (x+5)\,3^{2x-1} = $

$\underset{\substack{\uparrow \\ \text{LIMITNÍ} \\ \text{PŘECHOD}}}{=} +\infty \cdot 3^{+\infty} = +\infty \cdot +\infty$

$= +\infty$

B) $\lim\limits_{x\to-\infty} (x+5)\,3^{2x-1} \underset{\substack{\uparrow \\ \text{LIM} \\ \text{PŘECH}}}{=} -\infty\ 3^{-\infty} = -\infty \cdot 0$ NEDEF VÝRAZ

2 MOŽNÉ POSTUPY

1) $\lim\limits_{x\to-\infty} \dfrac{\overset{\text{"}-\infty\text{"}}{(x+5)}}{\underset{\substack{\text{"}3^{+\infty}\text{"} \\ \text{"}+\infty\text{"}}}{3^{-2x+1}}} = 0$

"EXPONENCIÁLNÍ FUNKCE ROSTE" DO $+\infty$ RYCHLEJI NEŽ POLYNOM

2) $\lim\limits_{x\to-\infty} \dfrac{(x+5)}{3^{-2x+1}} \underset{\substack{L'H \\ \frac{\text{"}-\infty\text{"}}{\text{"}+\infty\text{"}}}}{=} \lim\limits_{x\to-\infty} \dfrac{1}{3^{-2x+1}(-2)} \underset{\substack{\uparrow \\ \text{LIM} \\ \text{PŘECH}}}{=} \dfrac{1}{3^{+\infty}\cdot(-2)} = \dfrac{1}{-\infty} =$

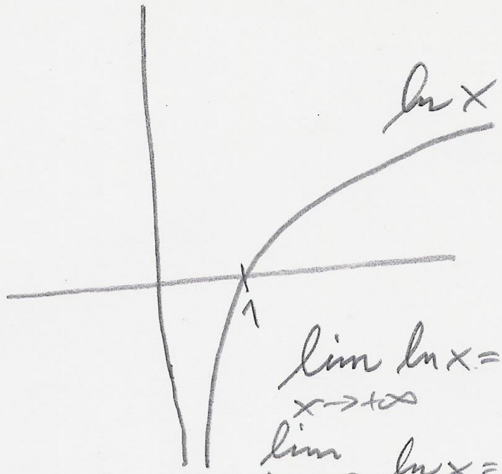
$= 0$

$\cdot\ln(3)$

$\left(3^{-2x+1}\right)' = 3^{-2x+1}\ln(3)\cdot(-2)$

$\boxed{Př}$ $f(x) = \dfrac{\ln x}{x-1}$

$D_f = (0;1) \cup (1;+\infty)$



$\lim_{x \to +\infty} \ln x = +\infty$

$\lim_{x \to 0_+} \ln x = -\infty$

A) $\lim_{x \to +\infty} \dfrac{\ln x}{x-1} \underset{\substack{\uparrow \\ \text{LIM} \\ \text{PŘECHOD}}}{=} \dfrac{+\infty}{+\infty}$

2 POSTUPY:

α) $\lim_{x \to +\infty} \dfrac{\ln x}{x-1} = 0$

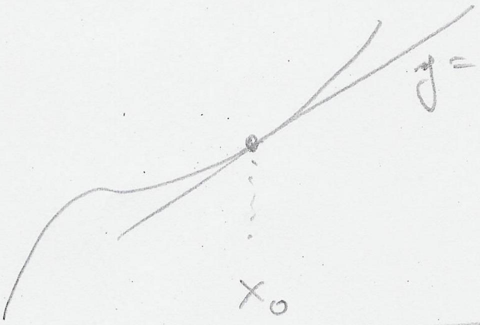
"POLYNOM ROSTE DO $+\infty$ RYCHLEJI" NEŽ LOGARITMUS

β) $\lim_{x \to +\infty} \dfrac{\ln x}{x-1} \underset{\text{„}\frac{+\infty}{+\infty}\text{"}}{\overset{L'H}{=}} \lim_{x \to +\infty} \dfrac{\frac{1}{x}}{1} \underset{\substack{\uparrow \\ \text{LIM PŘECH}}}{=} \dfrac{1}{+\infty} = 0$

B) $\lim_{x \to 1} \dfrac{\ln x}{x-1} = \dfrac{0}{0}$ NEDEF VÝRAZ $\underset{\text{„}\frac{0}{0}\text{"}}{\overset{L'H}{=}} \lim_{x \to 1} \dfrac{\frac{1}{x}}{1} \underset{\substack{\uparrow \\ \text{LIM} \\ \text{PŘECHOD}}}{=} 1$

C) $\lim_{x \to 0_+} \dfrac{\ln x}{x-1} \underset{\substack{\uparrow \\ \text{LIM} \\ \text{PŘECH}}}{=} \dfrac{-\infty}{-1} = +\infty$

Tečna ke grafu funkce



$$y = f'(x_0)(x-x_0) + f(x_0)$$

Př $f(x) = x^3 - 12x + 10$

$$f'(x) = 3x^2 - 12$$

v bodě $x_0 = 0$: $f'(0) = -12$, $f(0) = 10$

$$y = -12(x-0) + 10 = -12x + 10$$

v bodě $x_0 = 2$: $f'(2) = 0$, $f(2) = -6$

$$y = 0 \cdot (x-2) - 6 = -6$$

Př $f(x) = \ln(x^2+1)$

$$f'(x) = \frac{2x}{x^2+1}$$

v bodě $x_0 = 1$, $f'(1) = 1$, $f(1) = \ln(2)$

$$y = 1 \cdot (x-1) + \ln(2) = x - 1 + \ln(2)$$

v bodě $x_0 = 0$, $f'(0) = 0$, $f(0) = 0$

$$y = 0$$

Př $f(x) = e^{1-x^2}$

$f'(x) = e^{1-x^2} \cdot (-2x)$

v bodě $x_0 = 1$, $f'(1) = -2$, $f(1) = 1$

$y = -2(x-1) + 1 = -2x + 3$

v bodě $x_0 = 0$, $f'(0) = 0$, $f(0) = e$

$y = e$

Př $f(x) = 1 - x^2$, najděte všechny body z D_f, ve které má tečna grafu f směrnici a

$a = 1$

$f'(x) = -2x$

SMĚRNICE V OBECNÉM BODĚ

ŘEŠÍME ROVNICI: $-2x = 1$

$\boxed{x = -\frac{1}{2}}$

Př $f(x) = 3x^2 - 4x$, $a = -8$

$f'(x) = 6x - 4$

$\rightsquigarrow 6x - 4 = -8$

$\boxed{x = -\frac{4}{6} = -\frac{2}{3}}$

Př $f(x) = -x^2 + 4x + 21$, $a = 6$

$f'(x) = -2x + 4$, $-2x + 4 = 6$

$\boxed{x = -1}$

Monotonie funkce

$f'(x) > 0 \quad \forall x \in I \Rightarrow f$ roste v I

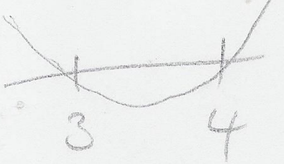
$f'(x) < 0 \quad \forall x \in I \Rightarrow f$ klesá v I

$f'(x_0) = 0 \; \dots \; x_0 \in D_f$ je stacionární bod

$\boxed{\text{Př}}$ Určete intervaly monotonie, souřadnice lok extrémů

$$f(x) = \frac{x^3}{3} - \frac{7x^2}{2} + 12x + 5$$

$$f'(x) = x^2 - 7x + 12 = (x-3)(x-4)$$



$(-\infty; 3)$	$(3; 4)$	$(4; +\infty)$
$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$
f roste	f klesá	f roste

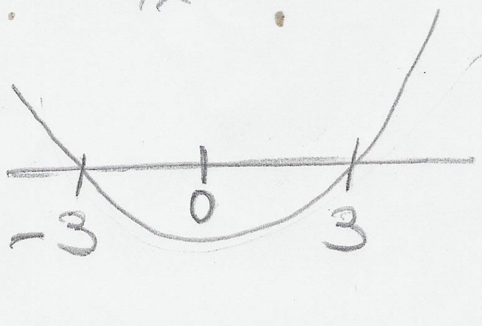
$\Downarrow$ $x_0 = 3$ je lok max

$\Downarrow$ $x_0 = 4$ je lok min

$\boxed{\text{Př}}$ $f(x) = \dfrac{x^2 - 10x + 9}{2x} \qquad D_f = \mathbb{R} \setminus \{0\}$

$$f'(x) = \frac{(2x-10)2x - (x^2 - 10x + 9)2}{4x^2} = \frac{4x^2 - \cancel{20x} - 2x^2 + \cancel{20x} - 18}{4x^2} =$$

$$= \frac{2(x^2-9)}{4x^2} = \frac{2(x-3)(x+3)}{4x^2}$$



$(-\infty;-3)$	$(-3;0)$	$(0;3)$	$(3;+\infty)$
$f'(x)>0$	$f'(x)<0$	$f'(x)<0$	$f'(x)>0$
f <u>roste</u>	f <u>klesá</u>	f <u>klesá</u>	f <u>roste</u>

$\Downarrow$ $x_0=-3$ je lokální <u>maximum</u>

$\Downarrow$ $x_0=3$ je lokální <u>minimum</u>

$\boxed{\text{Př}}$ $f(x)=e^{\frac{1+x}{1-x}}$, $D_f=\mathbb{R}\setminus\{1\}$

$$f'(x)=e^{\frac{1+x}{1-x}}\frac{(1-x)-(1+x)(-1)}{(1-x)^2}=\underbrace{e^{\frac{1+x}{1-x}}}_{\geq 0}\cdot\underbrace{\frac{2}{(1-x)^2}}_{\geq 0}$$

$(-\infty;0)$	$(0;+\infty)$
$f'(x)>0$	$f'(x)>0$
f roste	f roste

$\boxed{\text{Př}}$ $f(x)=e^{2x-x^2}$, $D_f=\mathbb{R}$

$$f'(x)=e^{2x-x^2}(2-2x)=\underbrace{2e^{2x-x^2}}_{\geq 0}(1-x)$$

$(-\infty;1)$	$(1;+\infty)$
$f'(x)>0$	$f'(x)<0$
f <u>roste</u>	f <u>klesá</u>

$\Downarrow$ $x_0=1$ je lok. <u>maximum</u>

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