$$\boxed{\text{CV 8}}$$

$\boxed{\text{Př}}$ $\lim\limits_{n\to\infty} \sqrt{n^2+3n} - \sqrt{n^2-2n} =$ ← JAK NEPOSTUPOVAT

$$= \lim_{n\to\infty} \sqrt{n^2}\left(\sqrt{1+\underbrace{\tfrac{3}{n}}_{\to 0}} - \sqrt{1-\underbrace{\tfrac{2}{n}}_{\to 0}}\right) =$$

$$= \lim_{n\to\infty} \sqrt{n^2}\left(\sqrt{1} - \sqrt{1}\right) = \lim_{n\to\infty} \sqrt{n^2} - \sqrt{n^2} = 0$$

SPRÁVNÝ POSTUP

$$\lim_{n\to\infty} \sqrt{n^2+3n} - \sqrt{n^2-2n} \cdot \frac{\sqrt{n^2+3n} + \sqrt{n^2+3n}}{\sqrt{n^2+3n} + \sqrt{n^2+3n}} =$$

$$= \lim_{n\to\infty} \frac{5n}{\sqrt{n^2+3n} + \sqrt{n^2+3n}} = \frac{5}{2}$$

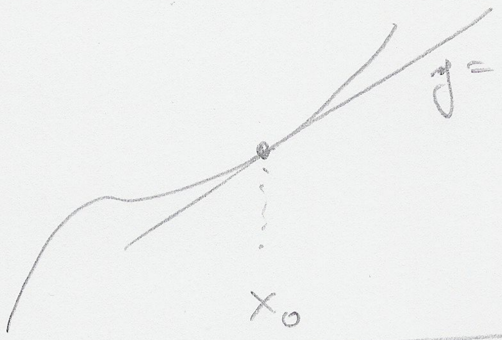
$\boxed{\text{Př}}$ $\lim\limits_{n\to\infty} \left(1+\underbrace{\tfrac{1}{n}}_{\to 0}\right)^n = \lim\limits_{n\to\infty} (1)^n = 1$

↑ ŠPATNÝ POSTUP

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$$

↑ ODVOZENO NA PŘEDNÁŠCE

Tečna ke grafu funkce



$$y = f'(x_0)(x - x_0) + f(x_0)$$

$\boxed{\text{Př}}$ $f(x) = x^3 - 12x + 10$

$$f'(x) = 3x^2 - 12$$

v bodě $x_0 = 0$: $f'(0) = -12$, $f(0) = 10$

$$y = -12(x - 0) + 10 = -12x + 10$$

v bodě $x_0 = 2$: $f'(2) = 0$, $f(2) = -6$

$$y = 0 \cdot (x - 2) - 6 = -6$$

$\boxed{\text{Př}}$ $f(x) = \ln(x^2 + 1)$

$$f'(x) = \frac{2x}{x^2 + 1}$$

v bodě $x_0 = 1$, $f'(1) = 1$, $f(1) = \ln(2)$

$$y = 1 \cdot (x - 1) + \ln(2) = x - 1 + \ln(2)$$

v bodě $x_0 = 0$, $f'(0) = 0$, $f(0) = 0$

$$y = 0$$

$\boxed{\text{Př}}$ $f(x) = e^{1-x^2}$

$f'(x) = e^{1-x^2} \cdot (-2x)$

v bodě $x_0 = 1$, $f'(1) = -2$, $f(1) = 1$

$y = -2(x-1) + 1 = -2x + 3$

v bodě $x_0 = 0$, $f'(0) = 0$, $f(0) = e$

$y = e$

$\boxed{\text{Př}}$ $f(x) = 1 - x^2$, najděte všechny body z D_f, pro které má tečna grafu f směrnici a

$a = 1$

$f'(x) = -2x$

SMĚRNICE V OBECNÉM BODĚ

ŘEŠÍME ROVNICI: $-2x = 1$

$\boxed{x = -\frac{1}{2}}$

$\boxed{\text{Př}}$ $f(x) = 3x^2 - 4x$, $a = -8$

$f'(x) = 6x - 4$

$\rightsquigarrow 6x - 4 = -8$

$\boxed{x = \frac{4}{6} = \frac{2}{3}}$

$\boxed{\text{Př}}$ $f(x) = -x^2 + 4x + 21$, $a = 6$

$f'(x) = -2x + 4$, $-2x + 4 = 6$

$\boxed{x = -1}$

L'HOSPITALOVO PRAVIDLO

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \text{„}\frac{0}{0}\text{"} \text{ nebo } \text{„}\frac{0}{\pm\infty}\text{"} \text{ nebo } \text{„}\frac{\pm\infty}{\pm\infty}\text{"}$$

$$\text{pak } \lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$$ MÁ-LI P.S. SMYSL

Př $$\lim_{x \to 1} \frac{x^2-1}{x-1} \overset{\cancel{x}}{\underset{L'H}{=}} \lim_{x \to 1} \frac{(x^2-1)'}{(x-1)'} = \lim_{x \to 1} \frac{2\cancel{x}}{\cancel{1}} = 2$$

„$\frac{0}{0}$"

Př $$\lim_{x \to +\infty} \frac{x^2-3}{1-3x^2} \overset{\cancel{x}}{\underset{L'H}{=}} \lim_{x \to \infty} \frac{2\cancel{x}}{-6\cancel{x}} = -\frac{1}{3}$$

„$\frac{\infty}{-\infty}$"

Př $$\lim_{x \to 1} \frac{\ln(x)}{1-x} \overset{\cancel{x}}{\underset{L'H}{=}} \lim_{x \to 1} \frac{\frac{1}{x}}{-1} = \frac{\frac{1}{1}}{-1} = -1$$

„$\frac{0}{0}$"

DOSADIT

Př $$\lim_{x \to 1} \frac{e^x - e}{x^2-1} \overset{\cancel{x}}{\underset{L'H}{=}} \lim_{x \to 1} \frac{e^x}{2x} = \frac{e^1}{2 \cdot 1} = \frac{e}{2}$$

„$\frac{0}{0}$"

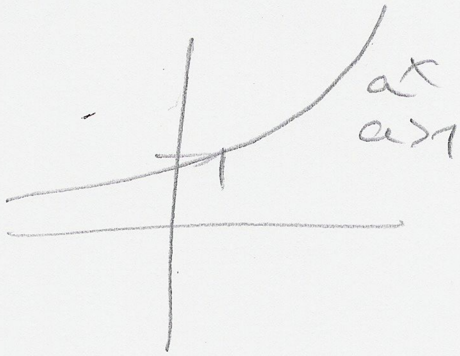
DOSADIT

LIMITY EXP LOG

známé limity:

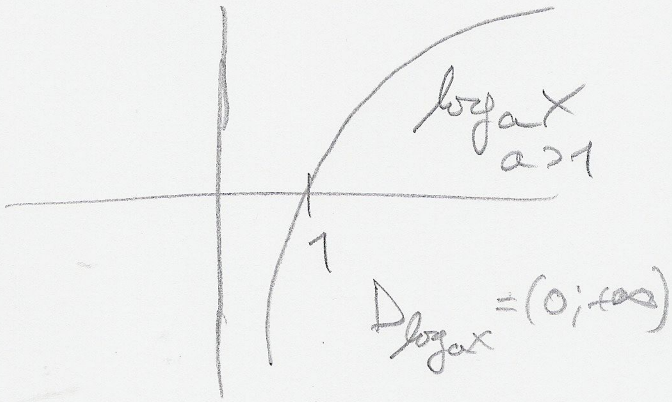
$$\lim_{x \to +\infty} a^x = +\infty, \quad a > 1$$

$$\lim_{x \to -\infty} a^x = 0, \quad a > 1$$



$$\lim_{x \to +\infty} \log_a x = +\infty$$

$$\lim_{x \to 0_+} \log_a x = -\infty$$



$\boxed{Př}$ $$\lim_{x \to +\infty} \left(\frac{4}{3}\right)^x = +\infty$$

$\boxed{Př}$ $$f(x) = \log(3x-8), \quad D_f = \left(\frac{8}{3}; +\infty\right)$$

$$3x - 8 > 0$$

$$\lim_{x \to +\infty} \log(3x-8) = +\infty$$

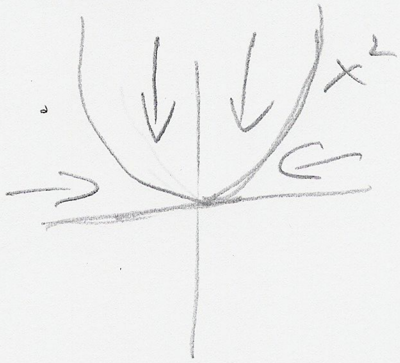
$$\lim_{x \to \frac{8}{3}+} \log(3x-8) = \lim_{x \to 0_+} \log x = -\infty$$

$\boxed{Př}$ $$f(x) = \log(x^2), \quad D_f = \mathbb{R} \setminus \{0\}$$

$$\lim_{x \to +\infty} \log(x^2) = \lim_{x \to +\infty} \log x = +\infty$$

$$\lim_{x \to -\infty} \log(x^2) = \lim_{x \to +\infty} \log x = +\infty$$

$$\lim_{x \to 0_+} \log(x^2) = \lim_{x \to 0_+} \log(x) = -\infty$$



$$\lim_{x \to 0_-} \log(x^2) = \lim_{x \to 0_+} \log(x) = -\infty$$

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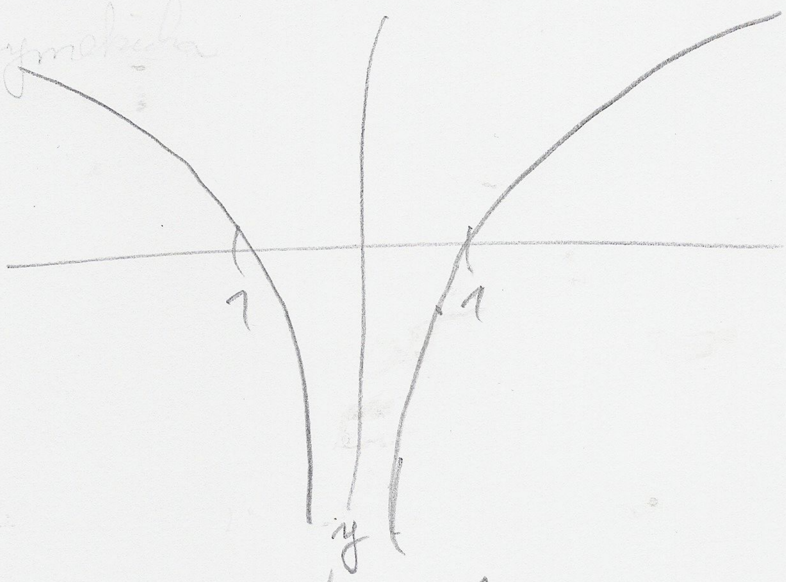
nákres $f(x)$



<u>SUDÁ FCE</u>: symetrická podle osy y

$f(-x) = f(x)$, $\log((-x)^2) = \log(x^2)$

<u>LICHÁ FCE</u>: středově symetrická podle počátku souřadnic $[0,0]$

$f(-x) = -f(x)$

$\boxed{P\pi}$

$$\lim_{x\to+\infty} e^{-x}(x^4-1) = \lim_{x\to+\infty} \frac{(x^4-1)}{e^x} \underset{\frac{\infty}{\infty}}{\overset{\text{L'H}}{=}} \lim_{x\to+\infty} \frac{4x^3}{e^x} \underset{\frac{\infty}{\infty}}{\overset{\text{L'H}}{=}}$$

$$\underset{\frac{\infty}{\infty}}{\overset{\text{L'H}}{=}} \lim \frac{4\cdot 3x^2}{e^x} \underset{\frac{\infty}{\infty}}{\overset{\text{L'H}}{=}} \lim \frac{4\cdot 3\cdot 2x}{e^x} \underset{\frac{\infty}{\infty}}{\overset{\text{L'H}}{=}}$$

$$= \lim_{x\to+\infty} \frac{4\cdot 3\cdot 2\cdot 1}{e^x} = 0$$

ZNÁMÉ LIMITY - POROVNÁVÁNÍ EXP, LOG, POLYNOM

$$\lim_{x\to+\infty} \frac{a^x}{x^n} = +\infty \;, \; a>1$$

$$\lim_{x\to+\infty} \frac{x^n}{a^x} = 0 \;, \; a>1$$

$$\lim_{x\to+\infty} \frac{\log_a x}{x^\alpha} = 0 \;, \; a>1, \; \alpha>0$$

$$\lim_{x\to+\infty} \frac{x^\alpha}{\log_a x} = +\infty \;, \; a>1, \; \alpha>0$$