Asymptoty v $+\infty$

A) $\lim\limits_{x\to+\infty} f(x) = \text{KONST}$ ... $f(x)$ má v $+\infty$ asymptotu $y = \text{KONST}$

B) $\lim\limits_{x\to+\infty} f(x) = \pm\infty$

1) $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = \pm\infty$ ... f nemá v $+\infty$ asymptotu

2) $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = 0$ .... f nemá v $+\infty$ asymptotu

3) $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = \text{KOSTANTA} = a \neq 0$

$\Rightarrow$ f má v $+\infty$ asymptotu danou $y = ax + b$,

kde $b = \lim\limits_{x\to+\infty} [f(x) - ax]$

((C) $\lim\limits_{x\to+\infty} f(x)$ NEEXISTUJE ... f nemá v $+\infty$ asymptotu))

$\boxed{Př}$ $f(x) = \frac{2x-3}{2x}$, $D_f = \mathbb{R} \setminus \{0\}$

$\lim_{x \to +\infty} \frac{2x-3}{2x} = 1$ ... $f(x)$ má v $+\infty$ asymptotu

$y = \frac{1}{2}$

$\lim_{x \to -\infty} \frac{2x-3}{2x} = 1$ .... $f(x)$ má v $-\infty$ asymptotu

$y = \frac{1}{2}$

$\lim_{x \to 0+} \frac{2x-3}{2x} = -\infty$, $\lim_{x \to 0-} \frac{2x-3}{2x} = +\infty$ ... $f(x)$ má v 0 asymptotu

$x = 0$

$\boxed{\text{Př}}$ $f(x) = \dfrac{x^2-10x+9}{2x}$ $\quad D_f = \mathbb{R}\setminus\{0\}$

$$\lim_{x\to+\infty} \frac{\frac{x^2-10x+9}{2x}}{x} = \lim_{x\to+\infty} \frac{x^2-10x+9}{2x^2} = \frac{1}{2} = a$$

$$\lim_{x\to+\infty} f(x) - ax = \lim_{x\to+\infty} \frac{x^2-10x+9}{2x} - \frac{1}{2}x =$$

$$= \lim_{x\to+\infty} \frac{x^2-10x+9-x^2}{2x} = \lim_{x\to+\infty} \frac{-10x+9}{2x} = -5$$

$f(x)$ má v $+\infty$ asymptotu $y = \frac{1}{2}x - 5$

$$\lim_{x\to-\infty} \frac{x^2-10x+9}{2x^2} = \frac{1}{2} = a$$

$$\lim_{x\to-\infty} \frac{x^2-10x+9}{2x} - \frac{1}{2}x = -5$$

$f(x)$ má v $+\infty$ asymptotu $y = \frac{1}{2}x - 5$

$$\lim_{x\to0_+} \frac{x^2-10x+9}{2x} = \frac{9}{0_+} = +\infty$$

$$\lim_{x\to0_-} \frac{x^2-10x+9}{2x} = \frac{9}{0_-} = -\infty$$

$f(x)$ má v 0 asymptotu $x = 0$

$\boxed{\text{Př}}$ Určete D_f + asymptoty

$f(x) = \sqrt{x^2-6x+10}$ $\qquad x^2-6x+10 \geq 0$

$D = 36 - 4\cdot 1\cdot 10 < 0$

$\Rightarrow$

$D_f = \mathbb{R}$

a) asymptota v $+\infty$

$$\lim_{x\to+\infty} \sqrt{x^2-6x+10} = \lim_{x\to+\infty} \sqrt{x^2\left(1-\frac{6}{x}+\frac{10}{x^2}\right)} =$$

$$= \lim_{x\to+\infty} |x| \sqrt{1-\frac{6}{x}+\frac{10}{x^2}} = \left(\lim_{x\to+\infty} x\right)\left(\lim_{x\to+\infty}\left(\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}\right)\right) =$$

$$= +\infty \cdot \sqrt{1} = +\infty$$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\sqrt{x^2-6x+10}}{x} = \lim_{x\to+\infty} \frac{\cancel{x}\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}}{\cancel{x}} =$$

$= \sqrt{1} = \underset{\neq 0}{1} = a$ ... f bude mít asymptotu, dopočteme b

$$\lim_{x\to+\infty} f(x) - ax = \lim_{x\to+\infty}\left(\sqrt{x^2-6x+10} - x\right) =$$

$$= \lim_{x\to+\infty}\left(\sqrt{x^2-6x+10} - \sqrt{x^2} \cdot \frac{\sqrt{x^2-6x+10}+\sqrt{x^2}}{\sqrt{x^2-6x+10}+\sqrt{x^2}}\right) =$$

$$= \lim_{x\to+\infty} \frac{-6x+10}{\sqrt{x^2-6x+10}+\sqrt{x^2}} = \lim_{x\to+\infty} \frac{-6x+10}{|x|\left(\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}\right)+|x|} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\left(-6+\frac{10}{x}\right)}{\cancel{x}\left[\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}+1\right]} = \frac{-6+0}{\sqrt{1}+1} = \frac{-6}{2} = -3$$

f má v $+\infty$ asymptotu $y = ax+b = x-3$

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b) asymptota v $-\infty$

$$\lim_{x\to-\infty} \sqrt{x^2-6x+10} = \lim_{x\to-\infty} \sqrt{x^2\left(1-\frac{6}{x}+\frac{10}{x^2}\right)} =$$

$$= \lim_{x\to-\infty} |x|\sqrt{1-\frac{6}{x}+\frac{10}{x^2}} = \left(\lim_{x\to-\infty}(-x)\right)\cdot\left(\lim_{x\to-\infty}\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}\right) =$$

$$= +\infty \cdot (\sqrt{1}) = +\infty$$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2-6x+10}}{x} = \lim_{x\to-\infty} \frac{(-\cancel{x})\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}}{\cancel{x}} =$$

$= -1\sqrt{1} = -1 = a$ ... f bude mít asymptotu, dopočteme b

x_0

$$\lim_{x\to-\infty} \sqrt{x^2-6x+10} - \underbrace{(-x)}_{|x|} = \lim_{x\to-\infty} \sqrt{x^2-6x+10} - \sqrt{x^2} =$$

$$= \lim_{x\to-\infty} \sqrt{x^2-6x+10} - \sqrt{x^2} \cdot \frac{\sqrt{x^2-6x+10}+\sqrt{x^2}}{\sqrt{x^2-6x+10}+\sqrt{x^2}} =$$

$$= \lim_{x\to-\infty} \frac{-6x+10}{\sqrt{x^2-6x+10}+\sqrt{x^2}} = \lim_{x\to-\infty} \frac{x\left(-6+\frac{10}{x}\right)}{|x|\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}+|x|} =$$

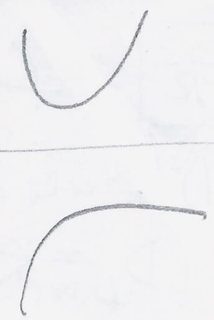
$$= \lim_{x\to-\infty} \frac{\cancel{x}\left(-6+\frac{10}{x}\right)}{-\cancel{x}\left[\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}+1\right]} = \frac{-6+0}{-1\left[\sqrt{1}+1\right]} = \frac{-6}{-2} = 3$$

. f má v $-\infty$ asymptotu $y = ax+b = -x+3$

Konvexnost / Konkávnost

$f''(x) > 0 \quad \forall x \in I \Rightarrow$ f je konVexní na I

$f''(x) < 0 \quad \forall x \in I \Rightarrow$ f je konkÁvní na I



inflexní bod: mění se konvexita s konkavitou