Průběh funkce

Př $f(x) = \dfrac{x^2+6x}{2-x}$

1) $D_f = \mathbb{R} \setminus \{2\}$

2) limity v krajních bodech D_f

$$\lim_{x\to+\infty} \frac{x^2+6x}{2-x} = \lim_{x\to+\infty} \frac{x^2\left(1+\frac{6}{x}\right)}{x\left(\frac{2}{x}-1\right)} = \lim_{x\to+\infty} x \cdot \lim_{x\to+\infty}\left(\frac{1+\frac{6}{x}}{-1+\frac{2}{x}}\right) =$$

$$= +\infty \cdot (-1) = \boxed{-\infty}$$

$$\lim_{x\to-\infty} \frac{x^2+6x}{2-x} = \dots = \lim_{x\to-\infty} x \cdot \lim_{x\to-\infty}\left(\frac{1+\frac{6}{x}}{-1+\frac{2}{x}}\right) = -\infty \cdot (-1) = \boxed{+\infty}$$

$$\lim_{x\to2^+} \frac{x^2+6x}{2-x} = \frac{16}{0-} = \boxed{-\infty}$$

$$\lim_{x\to2^-} \frac{x^2+6x}{2-x} = \frac{16}{0+} = \boxed{+\infty}$$

f(x) má asymptotu v 2

$x = 2$

3) průsečíky s osami

$P_y = [0;0]$, $P_{x_1} = [0,0]$, $P_{x_2} = [-6;0]$

$x^2+6x=0$

$x(x+6)$

$x_1 = 0$

$x_2 = -6$

4) asymptoty

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\cancel{x^2}(1+\frac{6}{x})}{\cancel{x^2}(\frac{2}{x}-1)} = -1 = a$$

$$\lim_{x\to+\infty} f(x)-ax = \lim_{x\to+\infty} \frac{x^2+6x}{2-x}+x =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^2}+6x+2x-\cancel{x^2}}{2-x} = \lim_{x\to+\infty} \frac{8x}{2-x} = -8 = b$$

f(x) má asymptotu v $+\infty$: $\boxed{y = -x-8}$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = -1 = a$$

$$\lim_{x\to-\infty} f(x)-ax = -8 = b$$

$\rightsquigarrow$ f(x) má asymptotu v $-\infty$ $y = -x-8$

5) první derivace

$$f'(x) = \frac{(2x+6)(2-x)-(x^2+6x)(-1)}{(2-x)^2} = \frac{4x-2x^2+12\cancel{-6x}+x^2\cancel{+6x}}{(2-x)^2} =$$

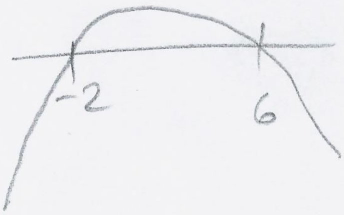
$$= \frac{-x^2+4x+12}{(2-x)^2} = \frac{-(x^2-4x-12)}{(2-x)^2} = \frac{-(x-6)(x+2)}{(2-x)^2}$$

6) monotonie

$(-\infty;-2)$	$(-2;2)$	$(2;6)$	$(6;+\infty)$
$f'(x)<0$	$f'(x)>0$	$f'(x)>0$	$f'(x)<0$
f klesá	f roste	f roste	f klesá

$\Downarrow$ $x_0=-2$ je lok min

$\Downarrow$ $x_0=6$ je lok max



7) extrémy

8) druhá derivace

$$f''(x) = \frac{(-2x+4)(2-x)^{\cancel{2}\,1} - (-x^2+4x+12)\,\cancel{2(2-x)}(-1)}{(2-x)^{\cancel{4}\,3}} =$$

$$= \frac{-\cancel{4x} + \cancel{2x^2} + 8 - \cancel{4x} - \cancel{2x^2} + \cancel{8x} + 24}{(2-x)^3} =$$

$$= \frac{32}{(2-x)^3} = \frac{32}{(2-x)(2-x)(2-x)}$$

9) konvexita/konkávnost

$+ + +$ | $- - -$

(+) 2 (−)

$(-\infty; 2)$	$(2; +\infty)$
$f''(x) > 0$	$f''(x) < 0$
konvexní	konkávní

$x_0 = 2$ není inflexní bod $\notin D_f$

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od) 7 dosazení extrémy

lok min $x_0 = -2$ $\rightarrow f(-2) = \frac{-8}{4} = -2$

lok max $x_0 = 6$ $\quad f(6) = \frac{36+36}{2-6} = -9-9 = -18$

10) graf

11) obor hodnot

$H_f = (-\infty; -18] \cup [-2; +\infty)$