Př2 Vyšetřování průběhu funkce

Průběžný test ZS 16/17 var C

$$f(x) = \frac{2x^2-2x-4}{x-3}$$

0) f není sudá ani lichá $[f(-x) \neq -f(x), \neq f(x)]$

1) $\boxed{D_f = \mathbb{R} \setminus \{3\}}$

2) limity v krajních bodech D_f

a) $$\lim_{x \to +\infty} \frac{2x^2-2x-4}{x-3} = \lim_{x \to +\infty} \frac{x^{\cancel{2}}\left(2-\frac{2}{x}-\frac{4}{x^2}\right)}{\cancel{x}\left(1-\frac{3}{x}\right)} =$$

$$= \lim_{x \to \infty} x \cdot \lim_{x \to +\infty} \frac{\left(2-\frac{2}{x}-\frac{4}{x^2}\right)}{\left(1-\frac{3}{x}\right)} = +\infty \cdot 2 = \boxed{+\infty}$$

b) $$\lim_{x \to -\infty} \frac{2x^2-2x-4}{x-3} = \lim_{x \to -\infty} \frac{x^{\cancel{2}}\left(2-\frac{2}{x}-\frac{4}{x^2}\right)}{\cancel{x}\left(1-\frac{3}{x}\right)} =$$

$$= \lim_{x \to -\infty} x \cdot \lim_{x \to -\infty} \frac{\left(2-\frac{2}{x}-\frac{4}{x^2}\right)}{\left(1-\frac{3}{x}\right)} = -\infty \cdot 2 = \boxed{-\infty}$$

c) $$\lim_{x \to 3^+} \frac{2x^2-2x-4}{x-3} = \frac{2 \cdot 9 - 6 - 4}{0_+} = \frac{8}{0_+} = \boxed{+\infty}$$

d) $$\lim_{x \to 3^-} \frac{2x^2-2x-4}{x-3} = \frac{2 \cdot 9 - 6 - 4}{0_-} = \frac{8}{0_-} = \boxed{-\infty}$$

$\Rightarrow$

$\Rightarrow$ f má svislou asymptotu v $x = 3$

3) průsečíky s osami

$P_y = \left[0; \frac{4}{3}\right]$

P_x: $\frac{2x^2-2x-4}{x-3} \overset{?}{=} 0$ $\qquad 2x^2-2x-4 \overset{?}{=} 0$

$2(x^2-x-2) = 2(x-2)(x+1) \Rightarrow$

$P_{x_1} = [2;0]$, $P_{x_2} = [-1;0]$

4) asymptoty

a) v $+\infty$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\frac{2x^2-2x-4}{x-3}}{x} = \lim_{x\to+\infty} \frac{2x^2-2x-4}{x(x-3)} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^2}\left(2-\frac{2}{x}-\frac{4}{x^2}\right)}{\cancel{x^2}\left(1-\frac{3}{x}\right)} = 2 = a$$

$$\lim_{x\to+\infty} \left[f(x) - ax\right] = \lim_{x\to+\infty} \left[\frac{2x^2-2x-4}{x-3} - 2x\right] =$$

$$= \lim_{x\to+\infty} \frac{2x^2-2x-4-2x(x-3)}{x-3} = \lim_{x\to+\infty} \frac{\cancel{2x^2}-2x-4-\cancel{2x^2}+6x}{x-3} =$$

$$= \lim_{x\to+\infty} \frac{4x-4}{x-3} = \lim_{x\to+\infty} \frac{\cancel{x}\left(4-\frac{4}{x}\right)}{\cancel{x}\left(1-\frac{3}{x}\right)} = \frac{4}{1} = 4 = b$$

$$\boxed{\begin{array}{l}\text{f má asymptotu v } +\infty \\ y = ax + b = 2x+4\end{array}}$$

b) v $-\infty$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \left[\begin{array}{l}\text{zcela analogický} \\ \text{postup jako v } +\infty\end{array}\right] = 2 = a$$

$$\lim_{x\to-\infty} \left[f(x) - ax\right] = \left[\qquad\right] = 4 = b$$

$$\boxed{\begin{array}{l}\text{f má asymptotu v } -\infty \\ y = ax + b = 2x+4\end{array}}$$

5) první derivace

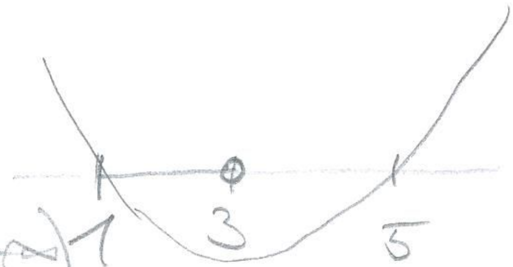
$$f'(x) = \frac{(4x-2)(x-3) - (2x^2-2x-4)(1)}{(x-3)^2} =$$

$$= \frac{4x^2 - 12x \cancel{-2x} + 6 - 2x^2 \cancel{+2x} + 4}{(x-3)^2} =$$

$$= \frac{2x^2 - 12x + 10}{(x-3)^2} = \frac{2(x^2-6x+5)}{(x-3)^2} = \frac{2(x-5)(x-1)}{\underbrace{(x-3)^2}_{\geq 0}}$$

$$D_{f'} = \mathbb{R} \setminus \{3\}$$

6) monotonie



$(-\infty; 1)$	$(1; 3)$	$(3; 5)$	$(5; +\infty)$
$f'(x) > 0$	$f'(x) < 0$	$f'(x) < 0$	$f'(x) > 0$
f roste	f klesá	f klesá	f roste

$\Downarrow$ $1 \in D_f$ $\qquad$ $3 \notin D_f$ $\qquad$ $\Downarrow$ $5 \in D_f$

f má v bodě $x_0 = 1$ lokální maximum

$$f(1) = \frac{2-1-4}{1-3} = \frac{2-5}{-2} = \frac{3}{2}$$

f má v bodě $x_0 = 5$ lokální minimum

$$f(5) = \frac{2 \cdot 25 - 10 - 4}{5-3} = \frac{36}{2} = 18$$

7) lokální extrémy

8) druhá derivace

$$f''(x) = \frac{(4x-12)(x-3)^{\cancel{2}\,1} - (2x^2-12x+20)(2\cancel{(x-3)}\cdot(1))}{(x-3)^{\cancel{4}\,3}} =$$

$$= \frac{\cancel{4x^2} - 12x - 12x + 36 - \cancel{4x^2} + 24x - 20}{(x-3)^3} = \frac{16}{(x-3)(x-3)(x-3)}$$

$$D_{f''} = \mathbb{R} \setminus \{3\}$$

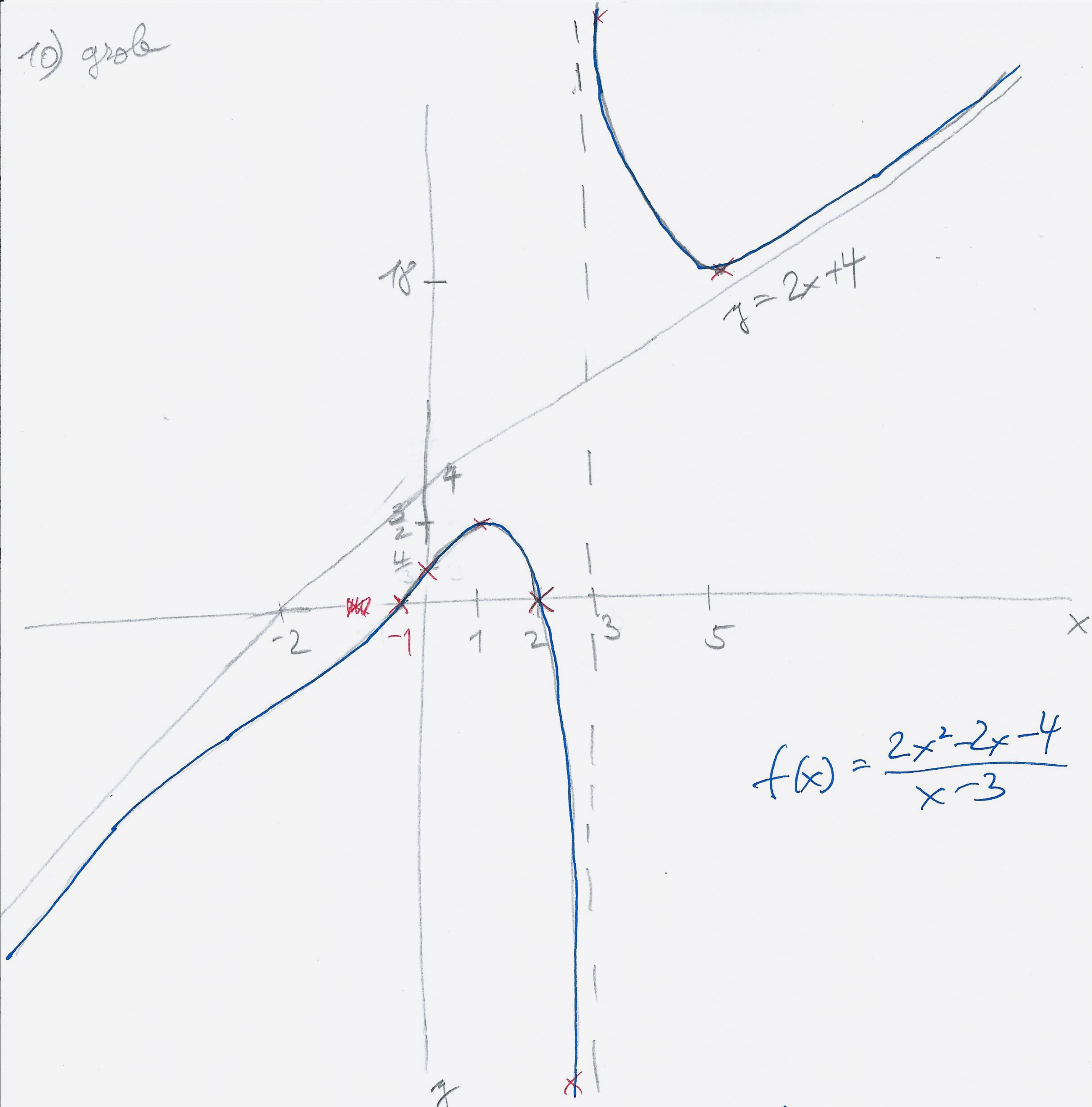
9) konvexnost/konkávnost

$$\begin{array}{ccc} - - - & & + + + \\ \hline \ominus & 3 & \oplus \end{array}$$

$(-\infty; 3)$	$(3; +\infty)$
$f''(x) < 0$	$f''(x) > 0$
f je konkávní	f je konvexní

$3 \notin D_f$

10) graf



11) obor hodnot $H_f = (-\infty; \frac{3}{2}] \cup [18; +\infty)$

12) globální extrémy ... funkce f nemá lokální extrémy