

A:

$$f(x) = \frac{\sqrt{4x^2 + x + 3}}{2x + 1}$$

$$Df: 4x^2 + x + 3 \geq 0$$

$$D = 1 - 4 \cdot (3) \cdot 4 < 0 \rightarrow \text{NETRA'KORĚN, } U \Rightarrow \downarrow \text{ PLATNĚ VĚDY}$$

$$2x + 1 \neq 0 \\ x \neq -\frac{1}{2} \Rightarrow D_f = \mathbb{R} \setminus \left\{ -\frac{1}{2} \right\} = \left( -\infty, -\frac{1}{2} \right) \cup \left( -\frac{1}{2}, +\infty \right)$$

$$A) \lim_{x \rightarrow +\infty} \frac{\sqrt{4x^2 + x + 3}}{2x + 1} = \lim_{x \rightarrow +\infty} \frac{|x| \sqrt{4 + \frac{1}{x} + \frac{3}{x^2}}}{x \left( 2 + \frac{1}{x} \right)} = \lim_{x \rightarrow +\infty} \frac{x \sqrt{4 + \frac{1}{x} + \frac{3}{x^2}}}{x \left( 2 + \frac{1}{x} \right)} = \\ = \frac{\sqrt{4}}{2} = 1$$

$$B) \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + x + 3}}{2x + 1} = \lim_{x \rightarrow -\infty} \frac{|x| \sqrt{4 + \frac{1}{x} + \frac{3}{x^2}}}{x \left( 2 + \frac{1}{x} \right)} = \lim_{x \rightarrow -\infty} \frac{(-x) \sqrt{4 + \frac{1}{x} + \frac{3}{x^2}}}{x \left( 2 + \frac{1}{x} \right)} = \\ = -\frac{\sqrt{4}}{2} = -1$$

$$C) \lim_{x \rightarrow -\frac{1}{2}^+} \frac{\sqrt{4x^2 + x + 3}}{2(x + \frac{1}{2})} = \frac{\sqrt{1 + (-\frac{1}{2}) + 3} > 0}{0^+} = +\infty$$

$$D) \lim_{x \rightarrow -\frac{1}{2}^-} \frac{\sqrt{4x^2 + x + 3}}{2(x + \frac{1}{2})} = \frac{\sqrt{1 + (-\frac{1}{2}) + 3} > 0}{0^-} = -\infty$$

$$\Rightarrow E) \lim_{x \rightarrow -\frac{1}{2}} \frac{\sqrt{4x^2 + x + 3}}{2(x + \frac{1}{2})} \text{ NEEXISTUJE}$$



B:

$$f(x) = \frac{x+2}{x^2-9} = \frac{x+2}{(x-3)(x+3)}$$

$$D_f = (-\infty; -3) \cup (-3; 3) \cup (3; +\infty)$$

$$A) \lim_{x \rightarrow +\infty} \frac{x+2}{x^2-9} = \lim_{x \rightarrow +\infty} \frac{x(1+\frac{2}{x})}{x^2(1-\frac{9}{x^2})} = \frac{1+0}{+\infty(1-0)} = \frac{1}{+\infty} = 0$$

$$B) \lim_{x \rightarrow -\infty} \frac{x+2}{x^2-9} = \lim_{x \rightarrow -\infty} \frac{x(1+\frac{2}{x})}{x^2(1-\frac{9}{x^2})} = \frac{1+0}{-\infty(1-0)} = \frac{1}{-\infty} = 0$$

$$C) \lim_{x \rightarrow -3_+} \frac{x+2}{(x-3)(x+3)} = \lim_{x \rightarrow -3_+} \frac{x+2}{x-3} \cdot \lim_{x \rightarrow -3_+} \frac{1}{x+3} = \frac{-1}{-6} \cdot \frac{1}{0_+} = +\infty$$

$$D) \lim_{x \rightarrow -3_-} \frac{x+2}{(x-3)(x+3)} = \lim_{x \rightarrow -3_-} \frac{x+2}{x-3} \cdot \lim_{x \rightarrow -3_-} \frac{1}{x+3} = \frac{-1}{-6} \cdot \frac{1}{0_-} = -\infty$$

$$E) C) / D) \Rightarrow \lim_{x \rightarrow -3} \frac{x+2}{x^2-9} \text{ NEEXISTUE}$$

$$F) \lim_{x \rightarrow +3_+} \frac{x+2}{(x-3)(x+3)} = \lim_{x \rightarrow +3_+} \frac{x+2}{x+3} \cdot \lim_{x \rightarrow +3_+} \frac{1}{x-3} = \frac{5}{6} \cdot \frac{1}{0_+} = +\infty$$

$$G) \lim_{x \rightarrow +3_-} \frac{x+2}{(x-3)(x+3)} = \lim_{x \rightarrow +3_-} \frac{x+2}{x+3} \cdot \lim_{x \rightarrow +3_-} \frac{1}{x-3} = \frac{5}{6} \cdot \frac{1}{0_-} = -\infty$$

$$H) F) / G) \Rightarrow \lim_{x \rightarrow +3} \frac{x+2}{x^2-9} \text{ NEEXISTUE}$$