Vyšetřete průběh funkce

ZÁVĚREČNÝ ~~TEST~~
LS2016/2017 VAR C

$$f(x) = \sqrt{x^2-10x+16} - 4$$

1) D_f:

$$x^2-10x+16 \geq 0$$
$$(x-2)(x-8) \geq 0$$

$$D_f = (-\infty; 2] \cup [8; +\infty)$$

2) sudost/lichost

$$f(2) = -4$$
$$f(-2) = \sqrt{4+20+16} - 4 = \underbrace{\sqrt{40}}_{\neq 8} - 4$$

$f(2) \neq f(-2) \Rightarrow f$ není sudá

$f(2) \neq -f(-2) \Rightarrow f$ není lichá

3) limity v krajních bodech D_f

$$\lim_{x\to+\infty} \sqrt{x^2-10x+16} - 4 = \lim_{x\to+\infty} x\sqrt{1-\frac{10}{x}+\frac{16}{x^2}} - 4 =$$

$$= (+\infty)\cdot\sqrt{1} - 4 = +\infty - 4 = +\infty$$

$$\lim_{x\to-\infty} \sqrt{x^2-10x+16} - 4 = \lim_{x\to-\infty} |x|\sqrt{1-\frac{10}{x}+\frac{16}{x^2}} - 4 =$$

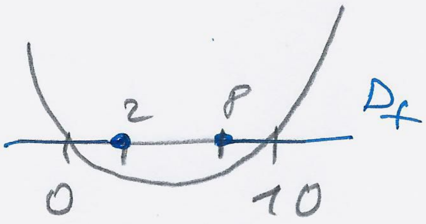
$$= \lim_{x\to-\infty} (-x)\sqrt{1-\frac{10}{x}+\frac{16}{x^2}} - 4 = (+\infty)\sqrt{1} - 4 = (+\infty) - 4 = +\infty$$

$$\lim_{x\to 2^-} \sqrt{x^2-10x+16} - 4 \underset{\text{dosadíme}}{=} -4$$

$$\lim_{x\to 8^+} \sqrt{x^2-10x+16} - 4 = -4$$

4) kladnost / zápornost

$$\sqrt{x^2-10x+16}-4 \le 0$$
$$\sqrt{x^2-10x+16} \le 4$$
$$x^2-10x+16 \le 16$$
$$x(x-10) \le 0$$



$f(x) \ge 0 \quad x \in (-\infty; 0] \cup [10; +\infty)$

$f(x) < 0 \quad x \in (0; 2) \cup (8; 10)$

5) Průsečíky

$P_y = [0; 0]$

$P_{x_1} = [0; 0]$, $P_{x_2} = [10; 0]$

6) Asymptoty

$\boxed{x \to +\infty}$

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\sqrt{x^2-10x+16}-4}{x} =$$

$$= \lim_{x \to +\infty} \frac{x\sqrt{1-\frac{10}{x}+\frac{16}{x^2}}-4}{x} = \lim_{x \to +\infty} \frac{\cancel{x}\sqrt{1-\frac{10}{x}+\frac{16}{x^2}}}{\cancel{x}} - \frac{4}{x} =$$

$$= \lim_{x \to +\infty} \sqrt{1-\frac{10}{x}+\frac{16}{x^2}} - \frac{4}{x} = \sqrt{1-0} = \sqrt{1} = 1 = a$$

$$\lim_{x \to +\infty} f(x) - ax = \lim_{x \to +\infty} \sqrt{x^2-10x+16}-4-x =$$

$$= \lim_{x \to +\infty} \left(\sqrt{x^2-10x+16}-4-x\right) \frac{\sqrt{x^2-10x+16}+(x+4)}{\sqrt{x^2-10x+16}+(x+4)} =$$

$$= \lim_{x\to+\infty} \frac{x^2-10x+16-(x^2+8x+16)}{\sqrt{x^2-10x+16}+(x+4)} = \lim_{x\to+\infty} \frac{-18\cancel{x}}{\cancel{x}\left(\sqrt{1-\frac{10}{x}+\frac{16}{x^2}}+1+\frac{4}{x}\right)} =$$

$$= \frac{-18}{\sqrt{1}+1} = -9 = b$$

f má v $+\infty$ šikmou asymptotu $y = x - 9$

$\boxed{v -\infty}$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2-10x+16}-4}{x} = \lim_{x\to-\infty} \frac{\cancel{(-x)}\sqrt{1-\frac{10}{x}+\frac{16}{x^2}}}{\cancel{x}} - \frac{4}{x} =$$

$$= -\sqrt{1} - 0 = -1 = a$$

$$\lim_{x\to-\infty} f(x) - ax = \lim_{x\to-\infty} \left(\sqrt{x^2-10x+16}-4+x\right)\left[\frac{\sqrt{x^2-10x+16}-(x-4)}{\sqrt{x^2-10x+16}-(x-4)}\right] =$$

$$= \lim_{x\to-\infty} \frac{x^2-10x+16-(x^2-8x+16)}{\sqrt{x^2-10x+16}-(x-4)} = \lim_{x\to-\infty} \frac{-2\cancel{x}}{-\cancel{x}\left[\sqrt{1-\frac{10}{x}+\frac{16}{x^2}}+1+\frac{4}{x}\right]} =$$

$$= \frac{-2}{-\left[\sqrt{1}+1\right]} = 1 = b$$

f má v $-\infty$ šikmou asymptotu $y = -x + 1$

---

jiný postup výpočtu limit

$$\lim_{x\to+\infty} \sqrt{x^2-10x+16} - x - 4 = \lim_{x\to+\infty} \left(\sqrt{x^2-10x+16}-x\right)\left[\frac{\sqrt{x^2-10x+16}+x}{\sqrt{x^2-10x+16}+x}\right] - 4$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\left(-10+\frac{16}{x}\right)}{\cancel{x}\left[\sqrt{1-\frac{10}{x}+\frac{16}{x^2}}+1\right]} - 4 = \frac{-10}{2} - 4 = -9$$

$$\lim_{x\to-\infty} \sqrt{x^2-10x+16} + x - 4 = \lim_{x\to-\infty} \left(\sqrt{x^2-10x+16}+x\right)\left[\frac{\sqrt{x^2-10x+16}-x}{\sqrt{x^2-10x+16}-x}\right] - 4 =$$

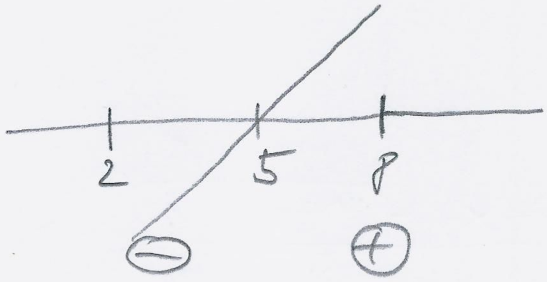
$$= \lim_{x\to-\infty} \frac{x\left(-10+\frac{16}{x}\right)}{-x\left[\sqrt{1-\frac{10}{x}+\frac{16}{x^2}}+1\right]} - 4 = \frac{-10}{-2} - 4 = 5-4 = 1$$

---

7) $f'(x) = \dfrac{2x-10}{2\sqrt{x^2-10x+16}} = \dfrac{x-5}{\sqrt{x^2-10x+16}}$ $D_{f'} = (-\infty;2) \cup (8;+\infty)$

8)+9) monotonie + lok. extrémy

$(-\infty;2)$	$(8;+\infty)$
$f'(x) < 0$	$f'(x) > 0$
f KLESÁ	f ROSTE



nemá lokální extrémy

10) $$f''(x) = \frac{\sqrt{x^2-10x+16} - \frac{(x-5)(x-5)}{\sqrt{x^2-10x+16}}}{x^2-10x+16} =$$

$$= \frac{\cancel{x^2-10x}+16-\cancel{x^2+10x}-25}{(x^2-10x+16)^{\frac{3}{2}}} = \frac{-9}{(x^2-10x+16)^{\frac{3}{2}}}$$

$D_{f''} = (-\infty;2) \cup (8;+\infty)$

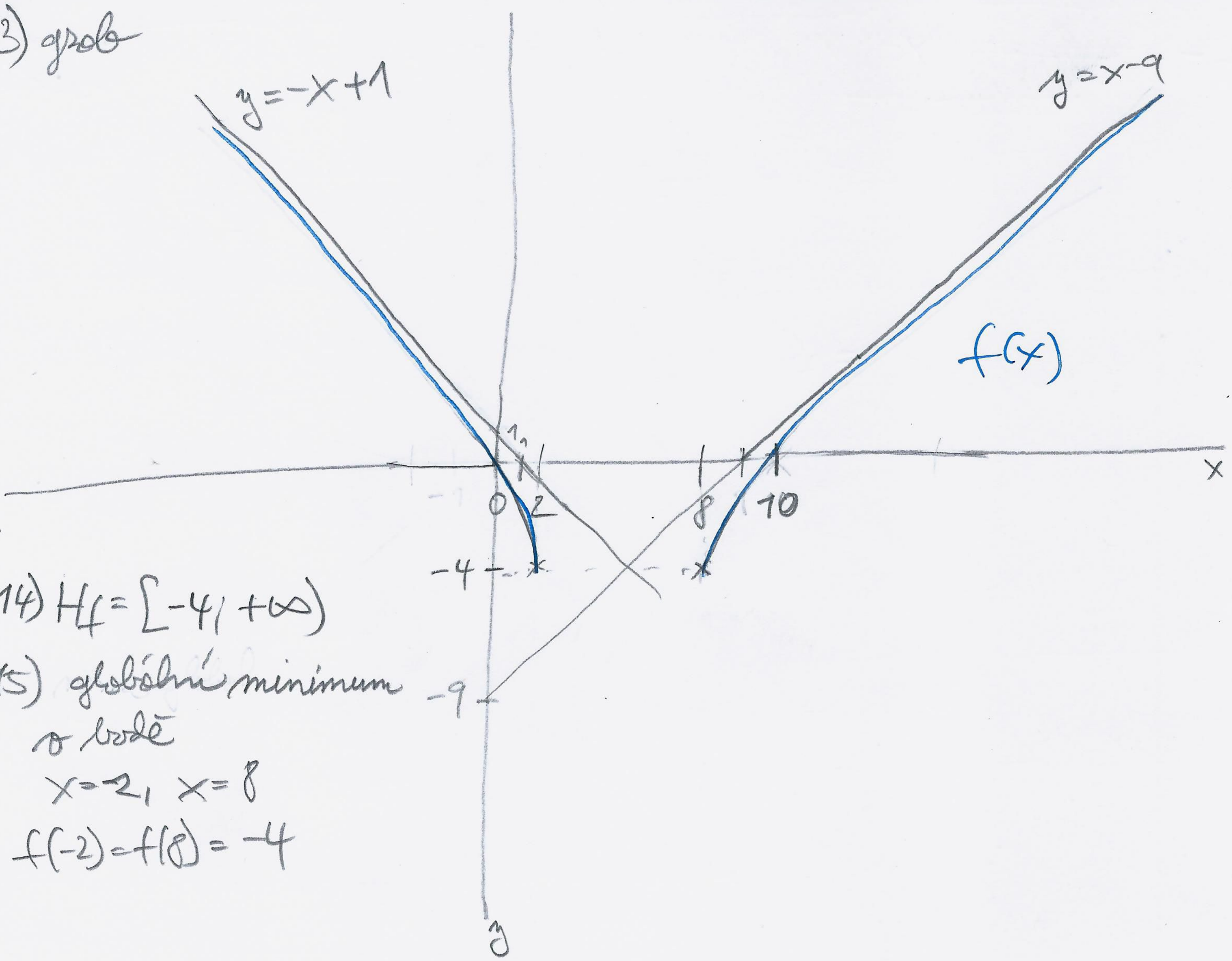
11)+12) konvex/konkáv + inflex body

$f''(x) < 0 \quad \forall x \in (-\infty;-2) \cup (8;+\infty)$

$\Rightarrow$ f je konkávní na celém D_f

$\Rightarrow$ inflexní body neexistují

13) graf



14) $H_f = [-4, +\infty)$

15) globální minimum v bodě
$x = -2,\ x = 8$
$f(-2) = f(8) = -4$